Antonine Physics A2

Topic 11 Electromagnetism and AC

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

Electricity and magnetism are inextricably linked. Indeed, the electric and magnetic forces were recognised very early on as different appearances of the same force. In this topic we see how magnetic fields arise from electric currents, and how a current carrying wire produces a force when it interacts with a magnetic field. This is at the heart of the electric motor. There are motors of all sizes from little motors in toys to giant machines that propel ships and pump water in pumped storage schemes.

We go on to look at generator effect where a wire moved through a magnetic field generates a magnetic field. We investigate Faraday's and Lenz's Laws. We then look at the operation of an alternating current generator.

Then we investigate transformers which enable the bulk transfer of electrical energy around the country in the National Grid.

There is, finally, some tutorials that discuss the reactive nature of capacitors and inductors in AC circuits. These are not examined in many syllabuses, but are in the SQA Advanced Higher Syllabus, the AQA Electronics Option, and WJEC Option A.

Topic 11

1. Magnetic Effect of an Electric Current

Tutorial 11.01 Magnetic Fields

All Syllabi

Contents

11.011 Magnetic Fields

You will be familiar with the basic notion of a **magnetic field**, in which **magnetic** materials experience a magnetic force. The force can come from an electric current, or a piece of magnetised material like a permanent magnet. Magnetic fields are different to other force fields because they are formed by **dipoles** (i.e. all magnets have a North and a South pole). This is shown in *Figure 1*.

Figure 1 Magnetic dipole

Electric fields are made using a single positive or negative point charge, and gravity is caused by point masses. You never get a magnetic **monopoles**. If you break up a magnet, you still get north and south poles (*Figure 2*).

Figure 2 A broken magnet consists of dipoles

This happens even if you grind the magnet down to atom sized particles.

It is worth revising some of the basic ideas that you will have come across in early secondary school (*Figure 3*).

- Magnetic fields can be shown by field lines, which go from North to South.
- The field lines in a strong magnetic field are more closely packed than in a weak field.

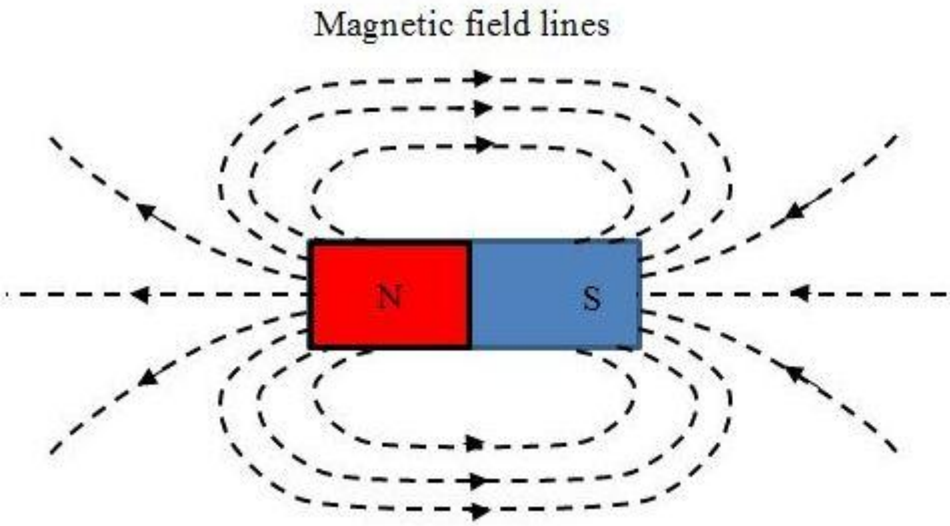


Figure 3 Magnetic Field of a bar magnet

- Unmagnetised materials are attracted to either pole.
- Like poles repel; unlike poles attract.
- In the Earth's magnetic field, the North pole will align itself to point to the North, if the magnet is allowed to swing freely.
- The Earth has a magnetic field like a bar magnet. Notice that the S-pole is under the North geographic pole. Be careful not to be confused by this (*Figure 4*).

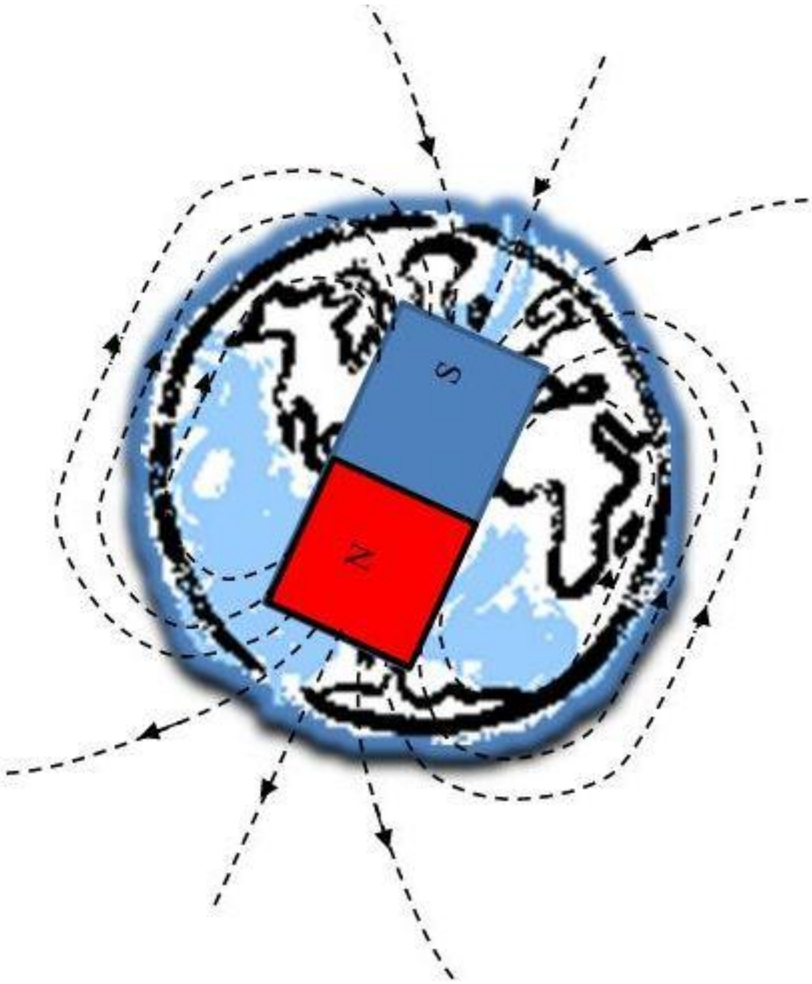
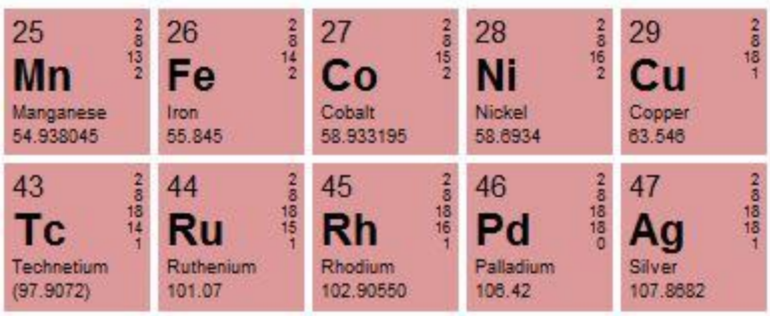


Figure 4 The Earth's magnetic field

- We never get single magnetic poles; if there is an N-pole, there must also be an S-pole to go with it.

Only iron, cobalt, and nickel and their alloys are magnetic (*Figure 5*).



25 Mn Manganese 54.938045 (2, 8, 13, 2)	26 Fe Iron 55.845 (2, 8, 14, 2)	27 Co Cobalt 58.933195 (2, 8, 15, 2)	28 Ni Nickel 58.6934 (2, 8, 16, 2)	29 Cu Copper 63.546 (2, 8, 18, 1)
43 Tc Technetium (97.9072) (2, 8, 18, 14, 1)	44 Ru Ruthenium 101.07 (2, 8, 18, 15, 1)	45 Rh Rhodium 102.90550 (2, 8, 18, 16, 1)	46 Pd Palladium 106.42 (2, 8, 18, 18, 0)	47 Ag Silver 107.8682 (2, 8, 18, 18, 1)

Figure 5 The only magnetic elements are next to each other in the Periodic Table

Note that these elements are next to each other in the periodic table. They are **transition elements**.

Some rare earth elements can be mixed to form alloys which can make powerful magnets. **Neodymium** magnets are made from an alloy consisting of neodymium, boron and iron ($Nd_2Fe_{14}B$) and are very powerful for their size.

We can show the fields of two magnets attracting (*Figure 6*).

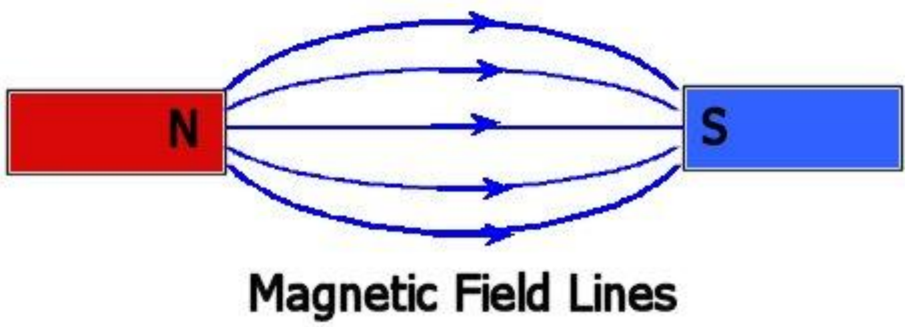


Figure 6 Field of unlike poles of magnets attracting

And repelling (*Figure 7*)

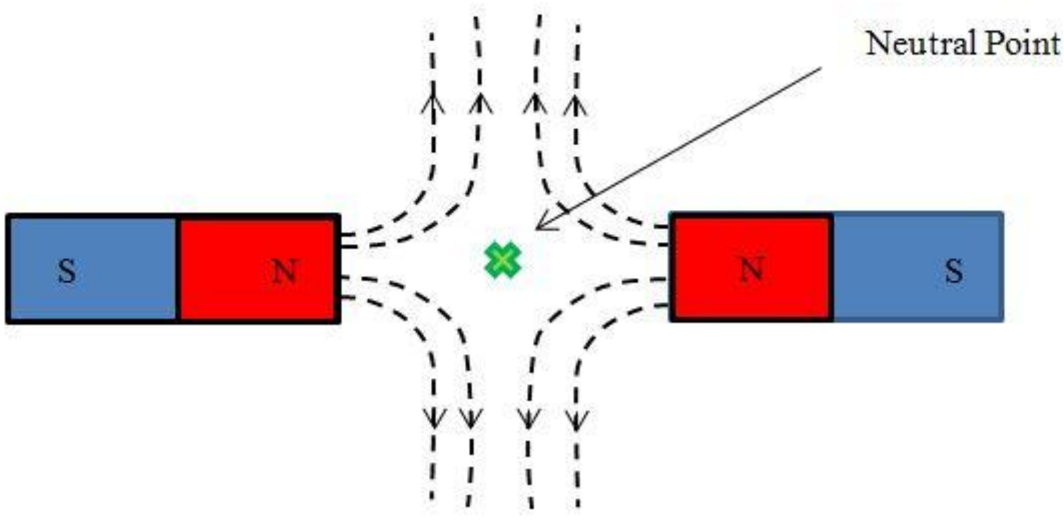


Figure 7 Magnetic field of like poles repelling

There is a **neutral point** where there is zero force.

11.012 Domain Theory of Magnetism

Magnets are thought to result from the action of tiny atomic magnets called **domains**. This can be explained by the movement of electrons that represent a tiny electric current that results in magnetism. In most materials, the currents cancel out. When a magnetic material is **unmagnetised**, the domains are **all jumbled up** (*Figure 8*).

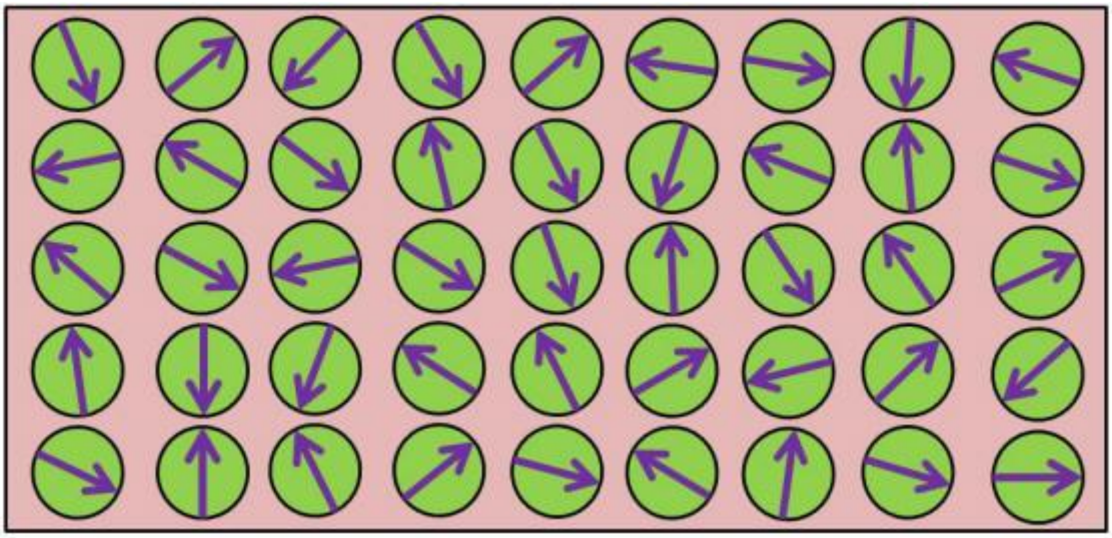


Figure 8 Unmagnetised magnetic material with randomly oriented domains

If **some** of the domains are **lined up**, then the material is **partially magnetised** (*Figure 9*).

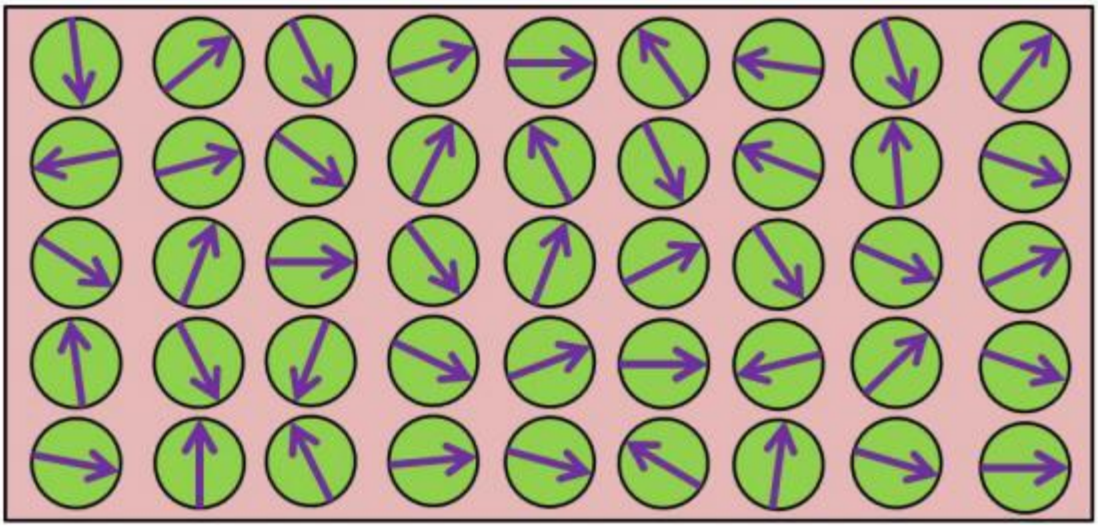


Figure 9 Partially magnetised material

If the domains are **fully** lined up, the magnet is **saturated** and cannot be magnetised further (*Figure 10*)

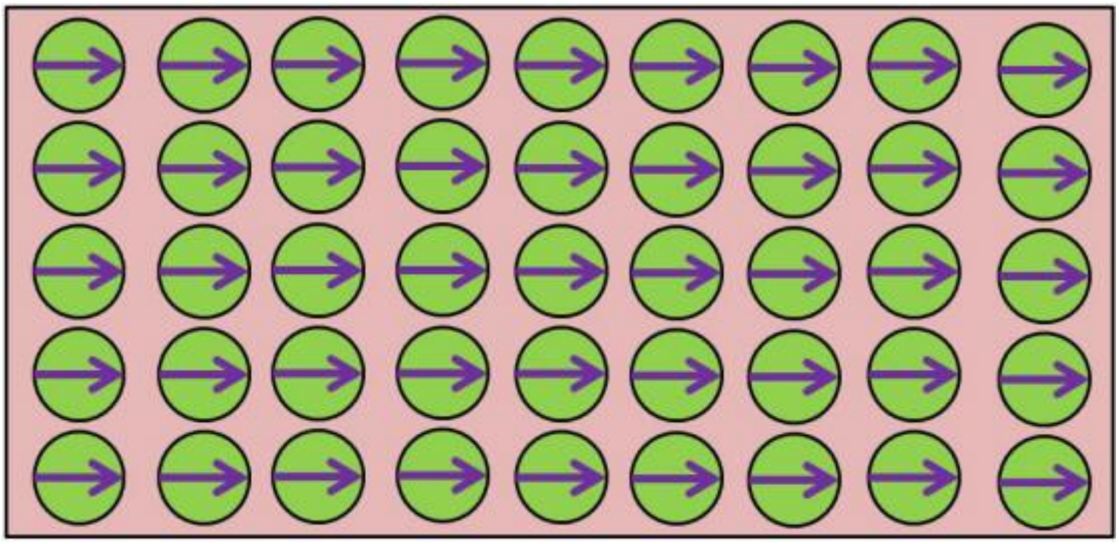


Figure 10 Saturated magnetic material

Some materials like **soft iron** lose their magnetism quickly. These are used for **temporary** magnets. **Permanent** or **hard** magnetic materials do not lose their magnetism.

11.013 Magnetic effect of an electric current

Electric currents are **always** associated with magnetic fields. The domains in a magnet are caused by the movement of electrons in shells. Electric currents **always** produce a magnetic field, even if the wire itself is not made of a magnetic material. The magnetic field of a single current carrying wire is like this (*Figure 11*).

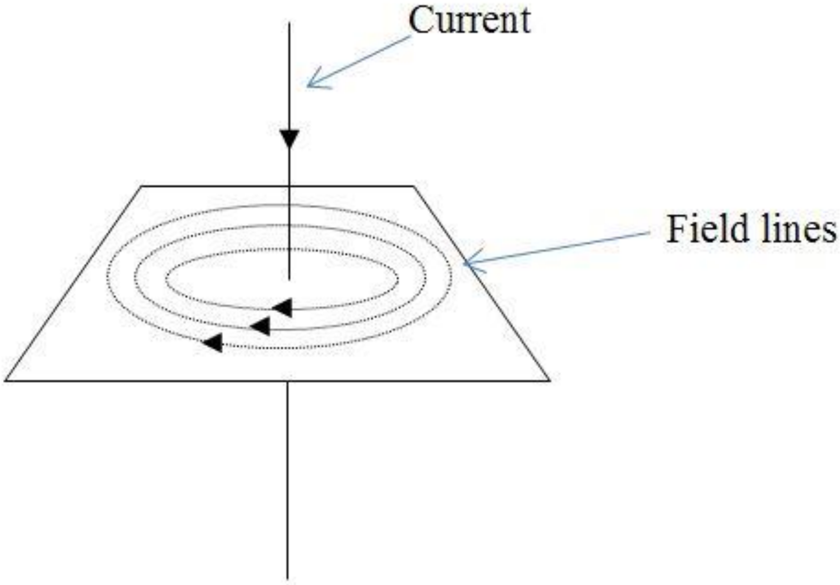


Figure 11 Magnetic field from a single current carrying wire

The direction of the current is determined by the **Screwdriver Rule** (*Figure 12*).

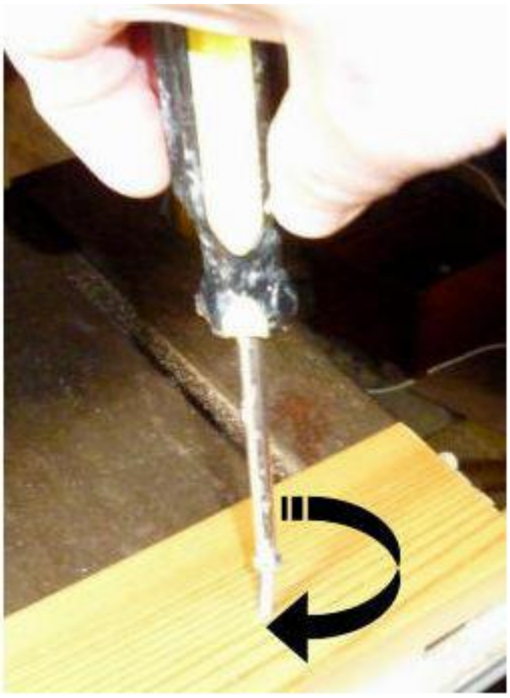


Figure 12 The Screwdriver Rule

The magnetic field strength depends on two factors:

- the current.
- the distance from the wire.

The relationship is:

$$B = \frac{\mu_0 I}{2\pi r}$$

.... *Equation 1*

Note that the magnetic field strength varies inversely with the distance, while gravity and electric fields vary inversely with the square of the distance.

Magnetic fields will interact. In the picture below two current carrying wires with the currents flowing in the **same** direction (*Figure 13*).

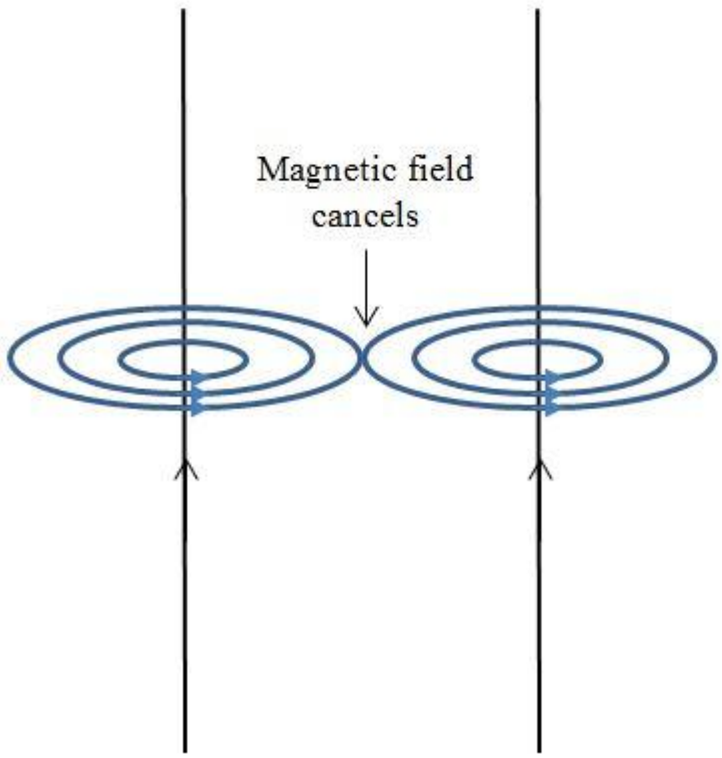


Figure 13 Magnetic fields made by two current-carrying wires in the same direction

There is a neutral point between the two wires where the magnetic field cancels (*Figure 14*).

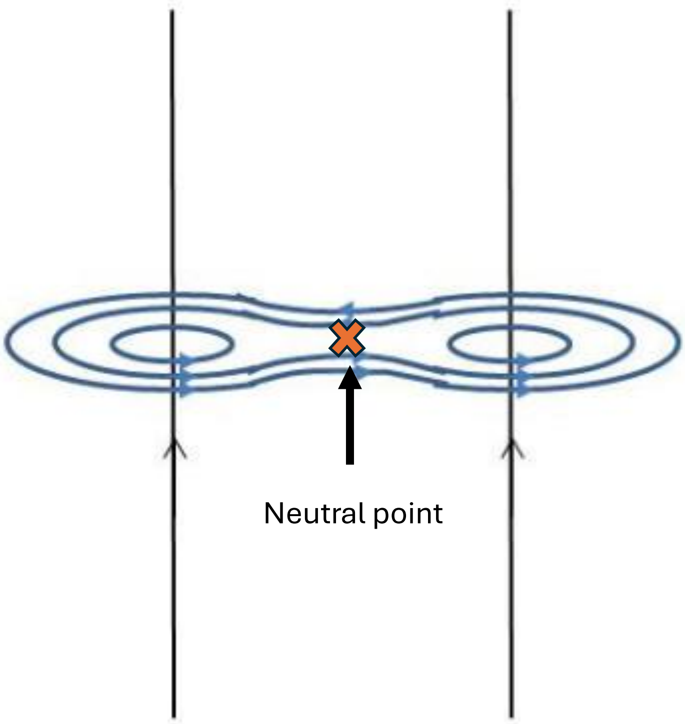


Figure 14 There is a neutral point

The two wires **attract**.

If the currents are in the opposite direction, there is **repulsion** (*Figure 15*)

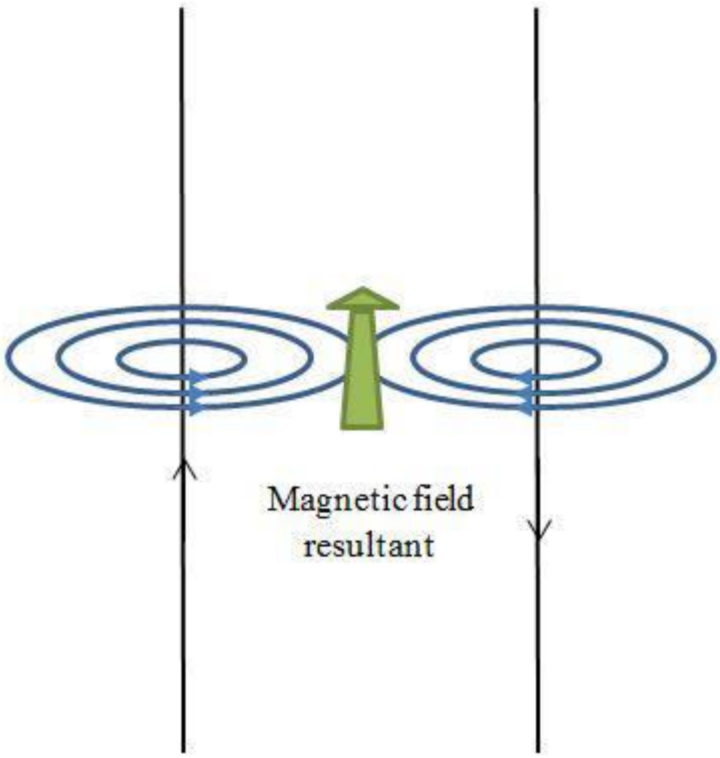


Figure 15 Currents in wires flowing in opposite directions

Notice that there is a **resultant** magnetic field.

This brings us onto a very important property of magnetic fields. Magnetic field strength or flux density is a **vector quantity**. The direction is important. Note also that magnetic fields are three-dimensional, although we show them as two dimensional as it's easier to do this. (I am not a very good artist.)

The SI unit for current is the Ampère (amp) is defined in these terms:

1 amp is the constant current, which when maintained in two parallel conductors of infinite length and negligible cross-section that are 1 metre apart in free space, produces a force of magnitude 2×10^{-7} newton per metre along their length.

The equation for the force between two parallel conductors carrying currents I_1 and I_2 respectively that are r metres apart is this:

$$\frac{F}{\Delta l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

.... *Equation 2*

The term $F/\Delta l$ is the **force per unit length**. The term μ_0 ("mu-nought") is a constant called the **permeability of free space**. The units for the permeability of free space are **Henry per metre** (H m^{-1})

$$\mu_0 = 4\pi \times 10^{-7}\ \text{H m}^{-1} = 1.257 \times 10^{-6}\ \text{H m}^{-1}$$

So, we can get a value for $F/\Delta l$:

$$F/\Delta l = (4\pi \times 10^{-7}\ \text{H m}^{-1} \times 1.0\ \text{A} \times 1.0\ \text{A}) \div (2 \times \pi \times 1.0\ \text{m})$$

This gives us:

$$F/\Delta l = \mathbf{2.0 \times 10^{-7}\ N\ m^{-1}}$$

11.014 Magnetic field of a Solenoid

A **solenoid** is a coil of wire usually wrapped around a former (*Figure 16*).

Figure 16 A solenoid

The magnetic field of a **solenoid** is like a **bar magnet** (*Figure 17*).

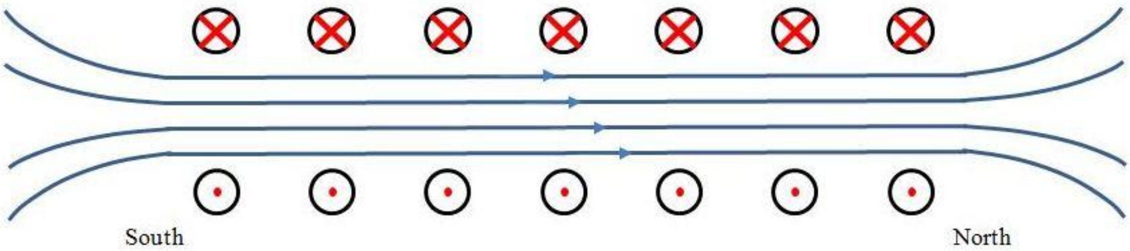


Figure 17 Magnetic field within a solenoid

The diagrams show a three dimensional picture in two dimensions. We can show the directions of the current more easily using dot and cross diagrams (*Figure 18*).

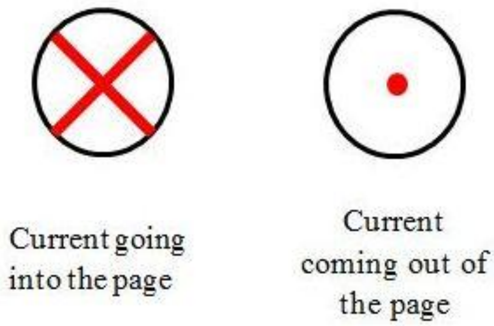


Figure 18 Dot and cross diagram

The current is shown vertical to the page (or screen - let's get up to date!). Outside the solenoid, the magnetic field is like this (*Figure 19*).

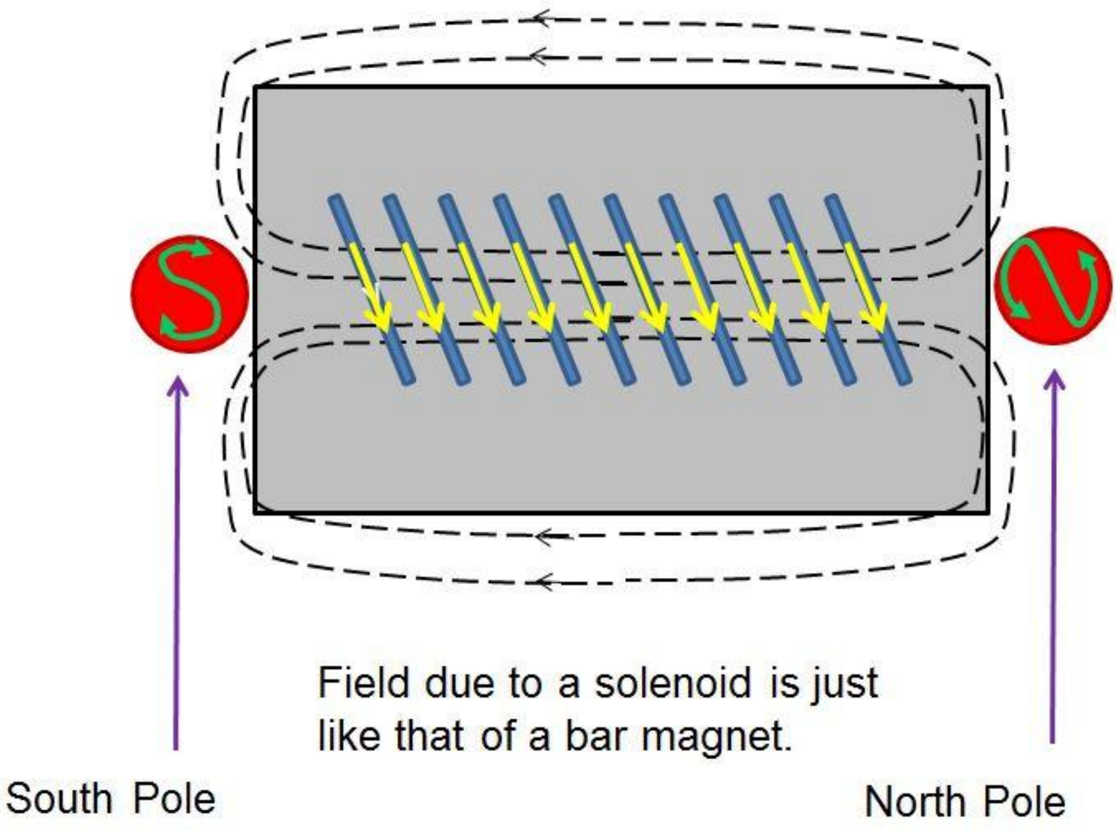


Figure 19 Magnetic field of a solenoid

If the current goes **clockwise**, we get a **south** pole. If the current goes **anticlockwise**, it's a **north** pole.

11.015 The Magnetic Field Strength of an Electric Current

(Welsh Board and Eduqas)

Consider a solenoid of n **turns per metre** which is carrying a current of I amps.

Note that the term n is **turns per metre**. So, you need to divide the total number of turns by the length.

If we measure the flux density in the solenoid well away from the ends, we find that:

$$B \propto I$$ ………… *Equation 3*

and

$$B \propto n$$ ……………… *Equation 4*

So, we can combine *Equations 3* and *4* to write:

$$B \propto nI$$ ……………. *Equation 5*

Strictly speaking, *Equations 3* to *5* should be described as **proportionalities**.

There is a constant of proportionality which is called the **permeability of free space**. It given the physics code μ_0 ("mu-nought" - the symbol 'μ' is 'mu', a Greek lower case letter 'm'). The units for the permeability of free space are **Henry per metre** (H m^{-1})

$$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1} = 1.257 \times 10^{-6} \text{ H m}^{-1}$$

Do not mix up the **permeability of free space** μ_0 with the **permittivity of free space** ε_0. The two words sound similar.

Strictly speaking, the permeability of free space applies to a vacuum. However, the value in air is very similar.

We can write an equation for this:

$$B = \mu_0 n I \quad \text{………… Equation 6}$$

Worked Example
A solenoid of length 2.5 cm has 200 turns. A current of 1.65 A flows through it. Calculate the resulting magnetic field strength. Give your answer to an appropriate number of significant figures.
Answer The number of turns per metre = 200 turns ÷ 0.025 = 8000 turns per metre. $B = 4\pi \times 10^{-7}$ H m^{-1} × 8000 m^{-1} × 1.65 A = 0.0166 T = **0.017 T** (= 17 mT) 2 significant figures as the length is to 2 s.f.

If we slide a bar of magnetic material into the central space of the solenoid, we have added a **core**. The magnetic field strength is increased by a factor μ_r ("mu-arr"), which is called the **relative permeability**. This factor is a **ratio**; therefore, there are no units. Our equation becomes:

$$B = \mu_0 \mu_r n I \quad \text{……………. Equation 7}$$

The relative permeability of pure iron is about 5000. Non-magnetic materials have a relative permeability of 1.

11.016 Force on a Current Carrying Wire

You will be familiar with the **motor effect**. If we put a current carrying wire in a magnetic field, we see that there is a force. The picture below shows a typical demonstration (*Figure 20*).

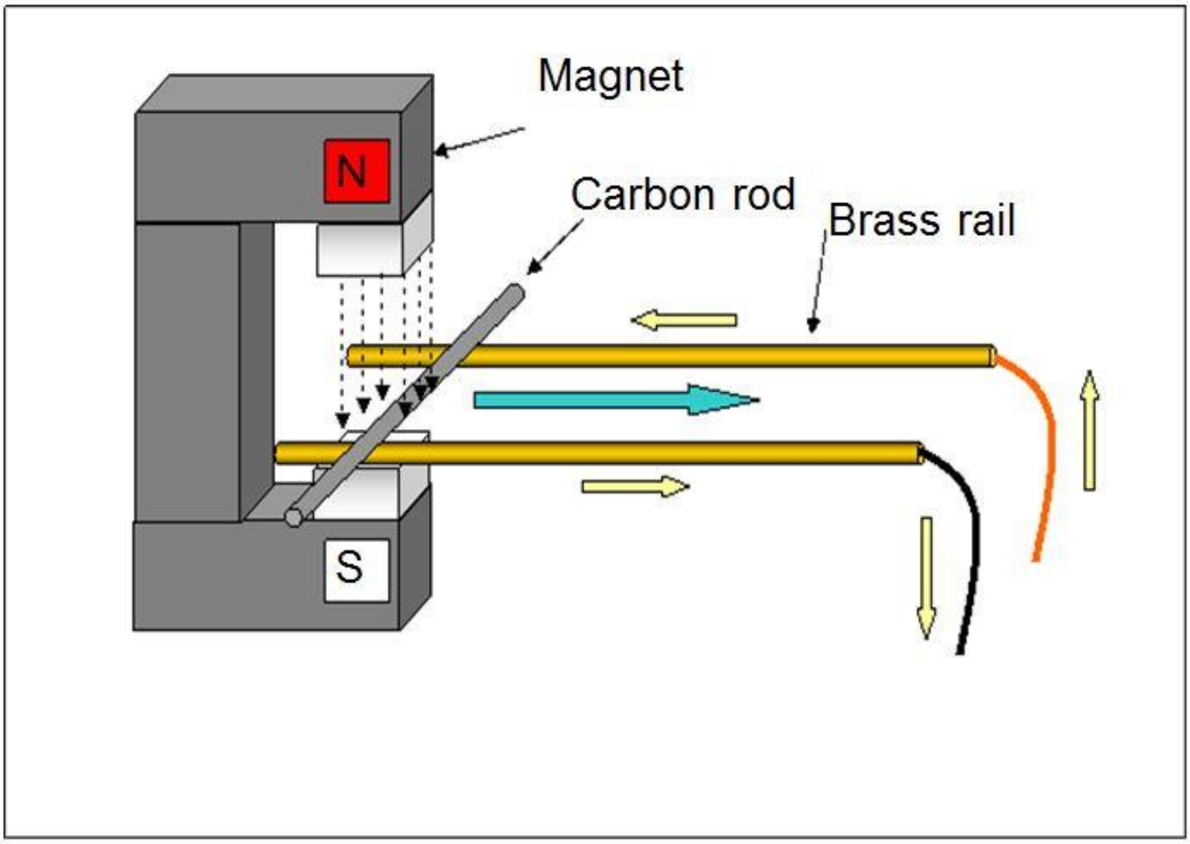


Figure 20 The motor effect

A carbon rod is placed in the magnetic field of a large permanent magnet. (You can't get anything less magnetic than carbon!) The brass rails connect the carbon rod to the power supply. When a current flows through the carbon rod in the directions indicated by the arrows, the rod experiences a force and moves from left to right. This shows that the current in the carbon rod makes a magnetic field that interacts with the magnetic field from the permanent magnet to produce a force.

If we turn the magnetic field so that it is parallel to the rails, the force is **zero** (*Figure 21*).

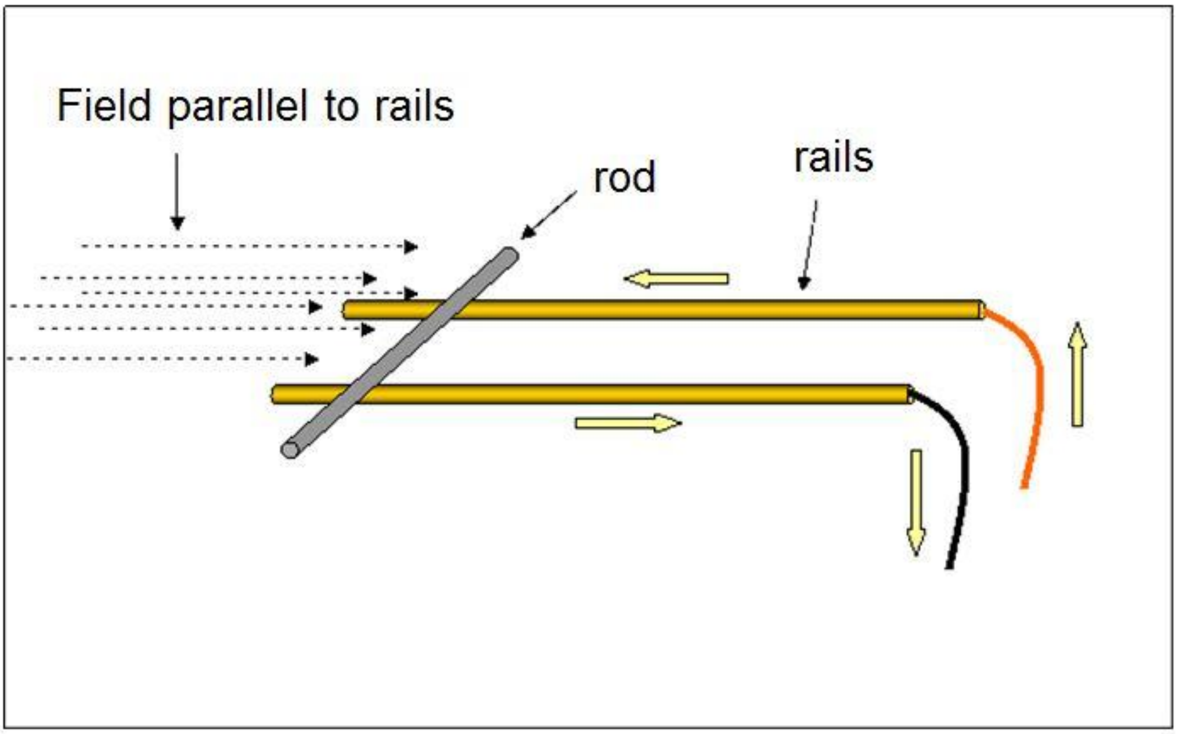


Figure 21 Motor effect when the field is parallel to the rails

Nothing happens.

11.017 Direction of the Force

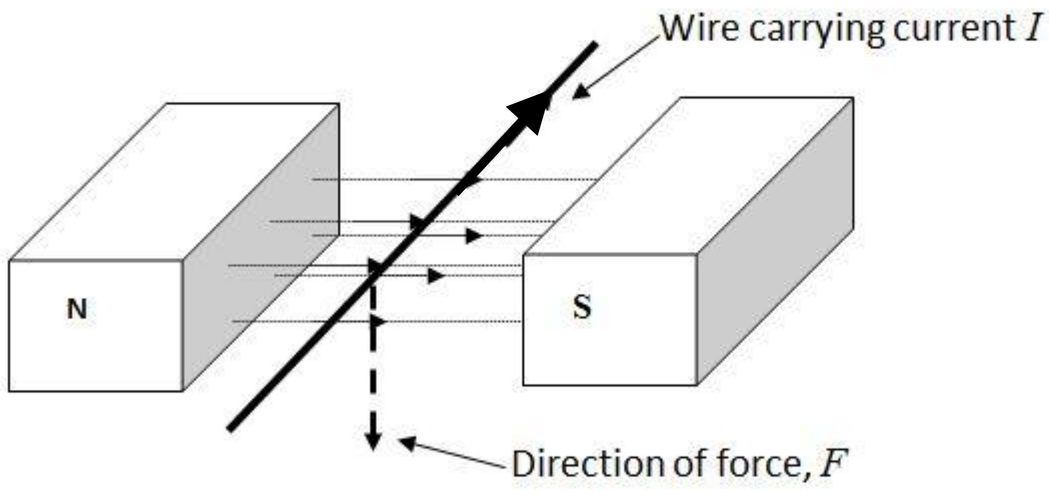


Figure 22 Force from a magnetic field on a current carrying wire

As we pass a current through the wire, there is a **force** that acts on the wire at **90°** to the direction of the magnetic field (*Figure 22*). This is given by **Fleming's Left Hand Rule** with which you will be familiar (*Figure 23*).

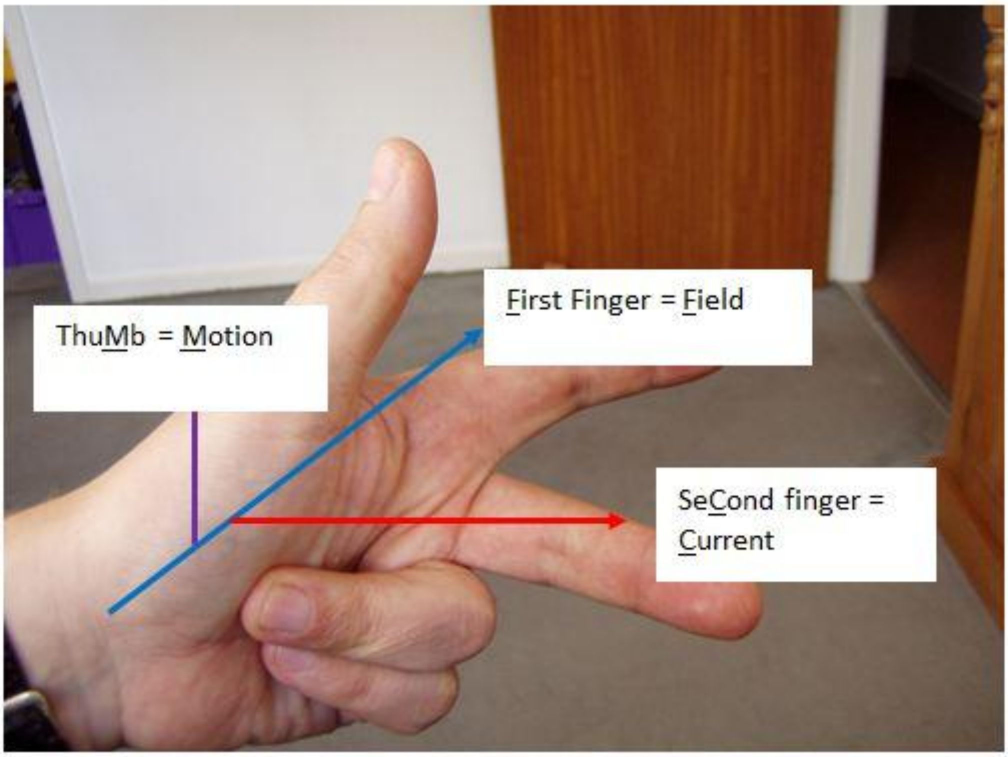


Figure 23 Fleming's Left Hand Rule

We can work out the force that is exerted on the wire quite simply. Experiment shows us that the force is proportional to:

- The current.
- The strength of the magnetic field.
- The length of wire within the magnetic field.

This is summed up in a simple formula:

$$F = BIl$$ ……………. *Equation 8*

[B – magnetic field strength (T); I – current (A); l – length (m)]

The term B is called the **magnetic field strength**, or the **flux density**, and is measured in **Tesla**, T. Flux density is a **vector** quantity. The magnetic flux density can be thought of as the concentration of field lines. We can increase the force by increasing any of the terms within the equation. If we coil up the wire, we increase its length within the magnetic field.

Worked Example A current of 8.5 A flowing through a magnetic field is found to exert a force of 0.275 N. The length of wire in the magnetic field is 5 cm. What is the value of the magnetic field?
Answer Formula first: $F = BIl$ $\Rightarrow B = \frac{F}{Il} = \frac{0.275\text{ N}}{8.5\text{ A} \times 0.05\text{ m}} = \mathbf{0.647\ T}$

If the field is at any angle other than 90 degrees, the formula takes this into account with the sine function:

$$F = BIl\sin\theta$$ ………... *Equation 9*

11.018 Required Practical - Force on a wire from a magnetic field

A simple experiment can be carried out to measure the force produced when a current flows through a magnetic field. The apparatus set up is straight-forward (*Figure 24*).

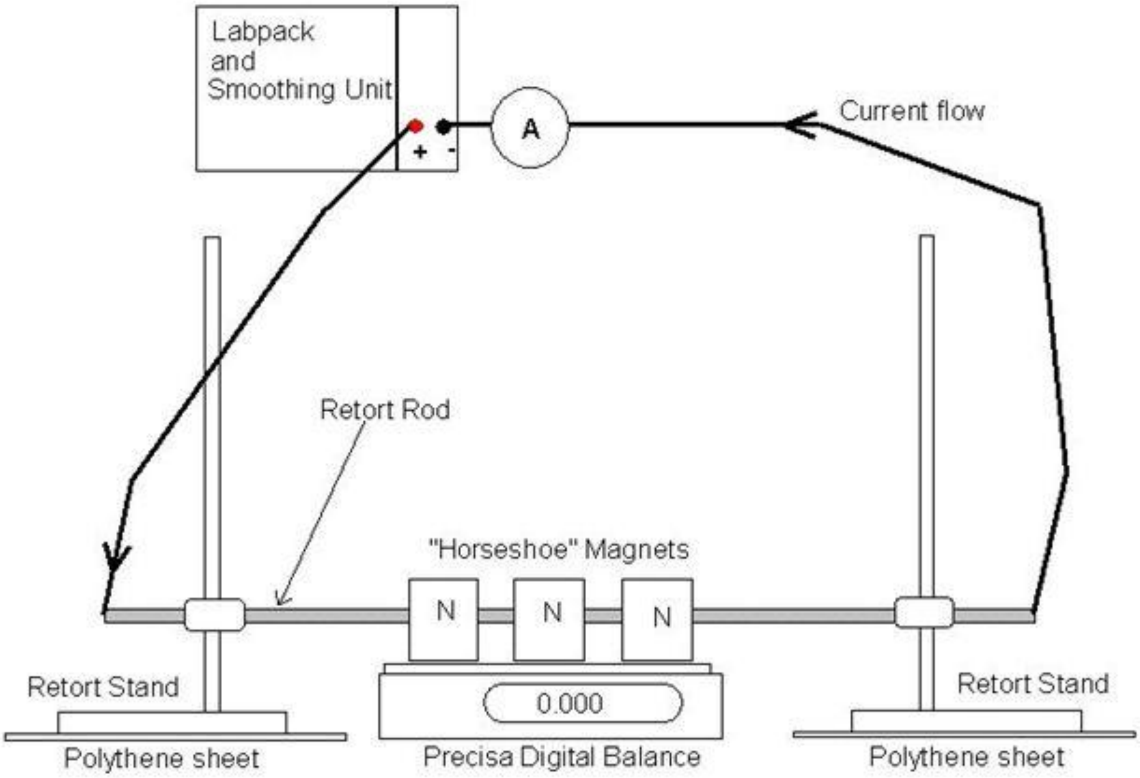


Figure 24 Measuring the magnetic force on a current carrying wire

In the diagram, a retort rod is used. A wire in a glass tube is also effective. The horseshoe magnets are from the motor kits that most schools and colleges have.

You need to:

- Measure the length of the wire that is in the magnetic field.
- Find the current that produces a measurable change in the reading of the balance.
- Measure the **change** in the reading.
- Convert the reading into **Newtons** (as the balance will give a reading in grams).
- Take at least seven readings from the minimum up to the maximum which the power supply will give.
- Do repeat readings and take averages.

You then need to plot a graph which will show **direct proportionality** (*Figure 25*).

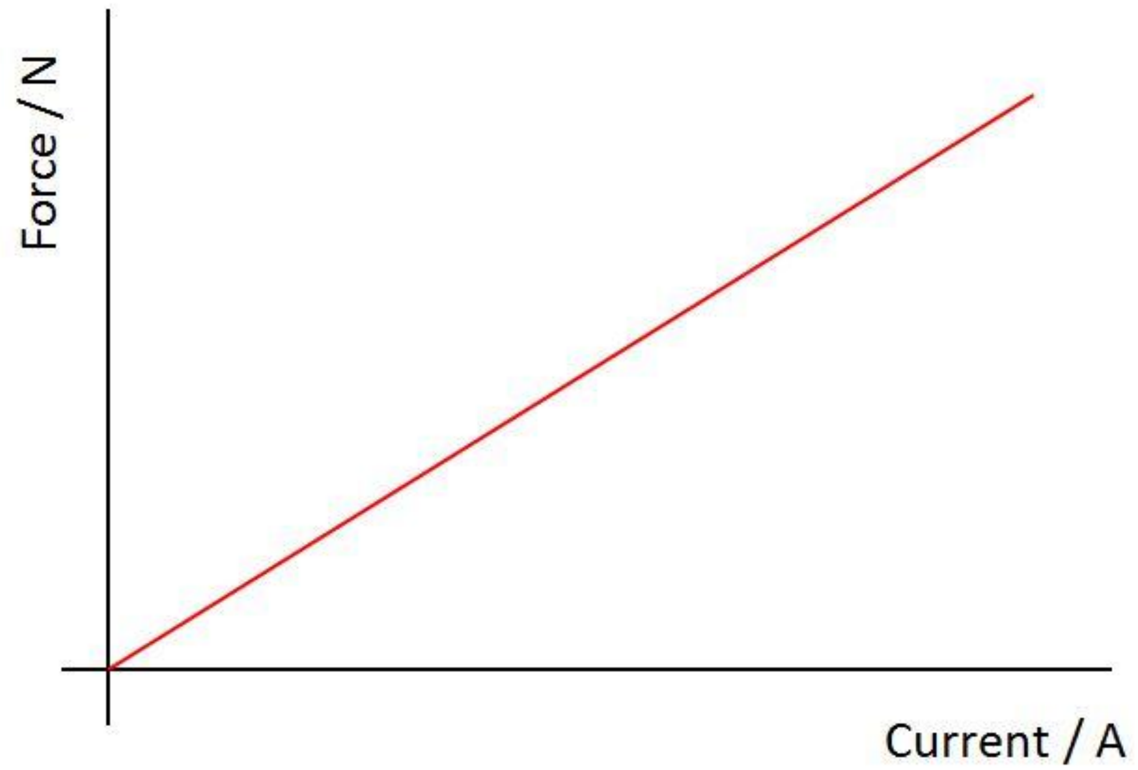


Figure 25 Graph of force against current showing direct proportionality

The magnetic field strength will come from the **gradient**.

$$\text{Gradient} = Bl$$

Therefore, we divide the gradient by the length to get a value for the magnetic flux density.

Tutorial 11.01 Questions

11.01.1

In a demonstration of the force on a current carrying wire, the length of wire in a magnetic field is 0.05 m. When a current of 2.5 A flows, a force of 0.01 N is shown. What is the magnetic field strength?

11.01.2

In a demonstration of the above equation, the length of wire in a magnetic field is 0.05 m. When a current of 2.5 A flows, a force of 0.01 N is shown. What is the magnetic field strength if the wire is at an angle of 35^{o} to the field?

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11.021 Coils in Fields

In the diagram below (*Figure 26*), we have a coil of wire, of n turns, suspended vertically in a magnetic field of strength B. It can rotate around the vertical axis.

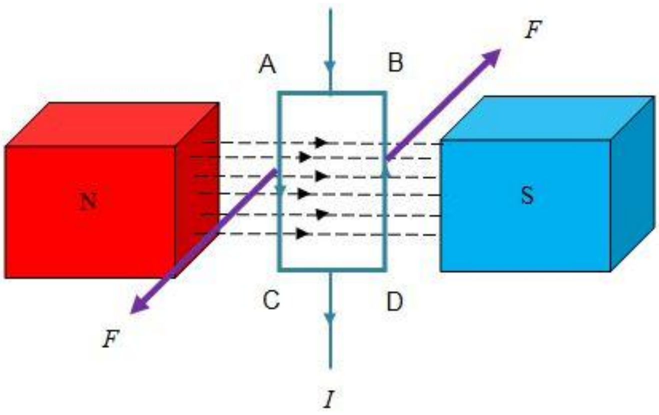

Figure 26 Coil in a magnetic field

A current *I* passes through the coil. We will look at the coil from above. If we set the coil parallel to the field, we will get this (*Figure 27*).

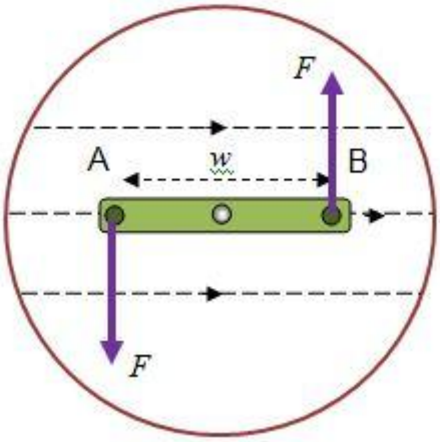

Figure 27 Looking down at a coil in a magnetic field

The w term is the width of the coil. The n term refers to the number of turns on the coil.

The sides AC and BD are vertical, and will experience a force:

$$F = n(BIl)$$ ………… *Equation 10*

The torque on the couple is given by:

$$\Gamma = Fw$$ ………… *Equation 11*

The strange looking symbol, Γ, which looks a bit like a gallows, is "Gamma", a Greek capital letter 'G'. It is often used as the Physics code for torque. In some texts, it is written as τ (tau).

Now the coil is at an angle θ (*Figure 28*).

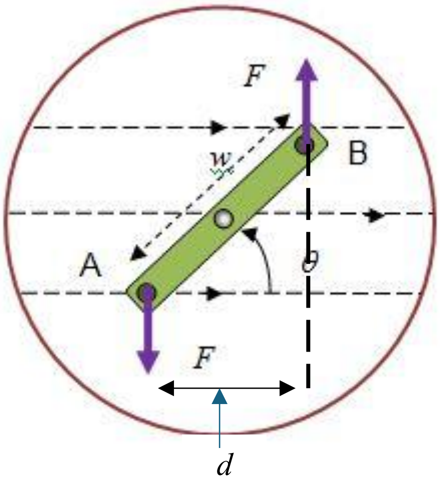

Figure 28 When the coil is at an angle

Because the force F is always at 90° to the magnetic field, the distance between the lines of action of the forces, d, reduces. Therefore, the resultant torque reduces. In the equation below we will call it Γ' ("gamma-prime") to distinguish it from the original torque, Γ.

$$\Gamma' = \underline{Fw} \cos\theta \quad \text{.........} \textit{ Equation 12}$$

$$= BIlnw \cos\theta \quad \text{..............} \textit{ Equation 13}$$

Since length × width, lw, gives area, A, we can now write:

$$\Gamma' = BAnI \cos\theta \quad \text{..........} \textit{ Equation 14}$$

When the coil is at 90° to the field, cos θ = 0. The magnitude of each of the two forces are still F N, but are in opposite directions, giving a **resultant force** of 0. Therefore, the torque is 0.

The bear-trap here is to think that the side AB is being acted on by the magnetic field. Remember that the force is acting on the vertical wires, AC and BD. We are looking down at the coil from above. The wires running between A and B (and C and D) are parallel with the magnetic field. Therefore, they experience no force.

10.022 Applications

The main application of this is with the **electric motor**. The **universal motor** is found in a variety of different household appliances. This one is from a vacuum cleaner (*Figure 29*)

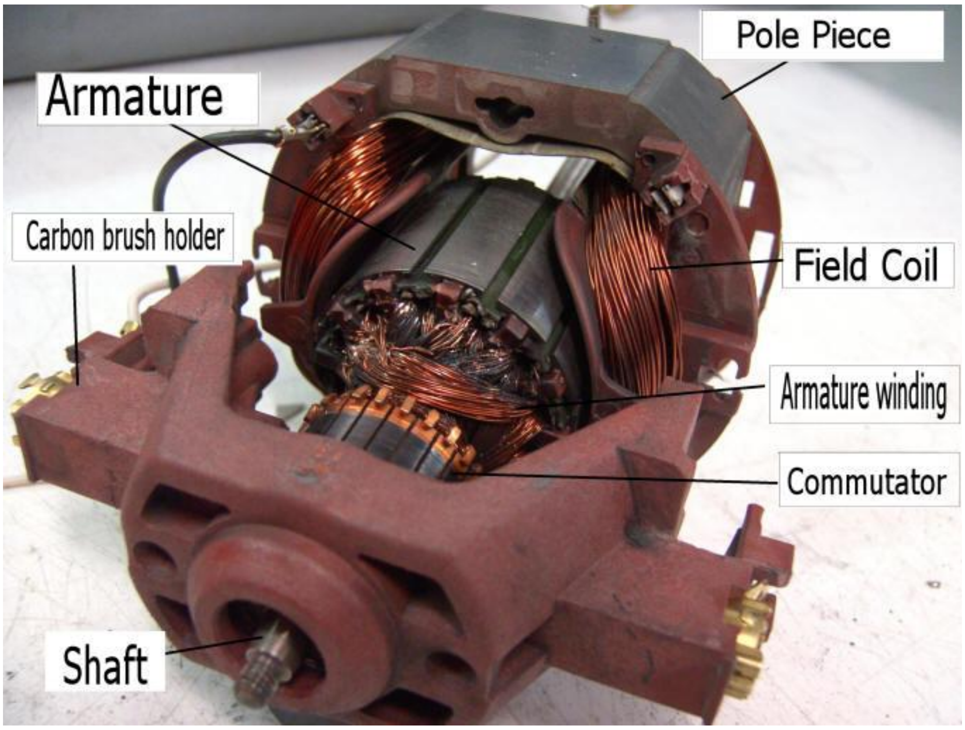

Figure 29 A universal motor (Photograph by Marrrci from Wikimedia Commons. Captions translated from the original German.)

Each of the coils is connected to the outside circuit by a split-ring **commutator** and spring-loaded carbon **brushes**.

Notice that the field magnets are curved. If the field is radial, the angle that the coil makes with the magnetic field is always constant at 0°.

In other words, the coil is always parallel to the magnetic field lines through which it is turning. Therefore, F remains constant, and the torque remains constant. This is shown in *Figure 30*.

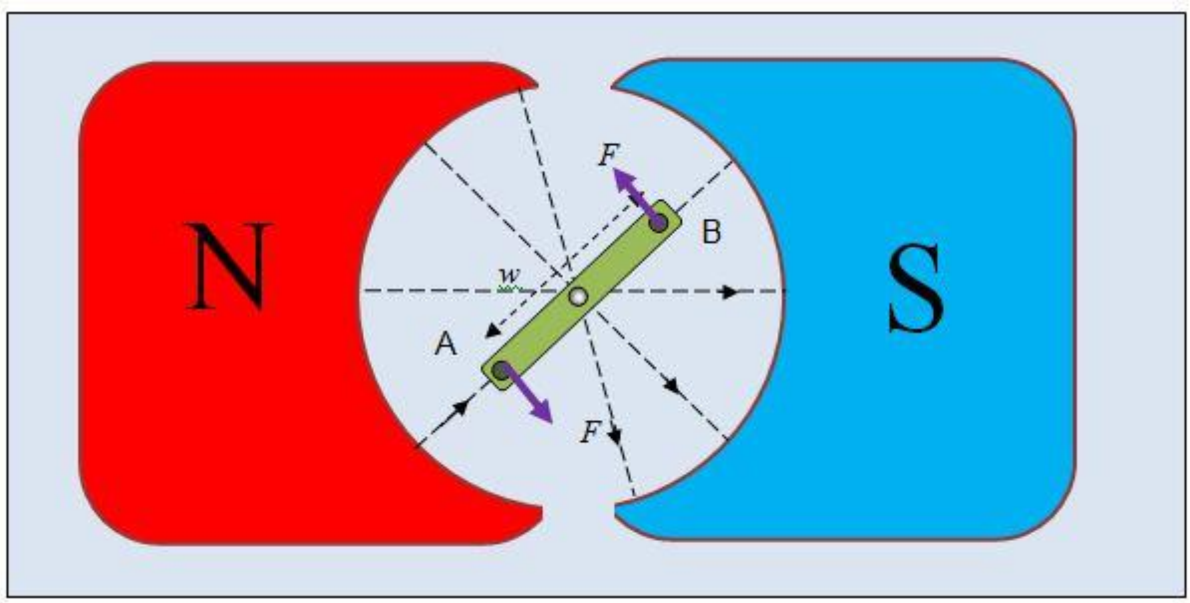

Figure 30 The pole pieces in most electric motors is radial

The universal motor can work on alternating current AC or on direct current DC. A 240 V AC motor can turn quite fast when connected to a 12 V DC supply. I know; I have tried it often. However, it is advisable to have a reverse-biased diode (*Figure 31*) across the terminals, as such a motor has a relatively high **inductance** (which we will look at later) and can give a high voltage spike then the motor is turned off. This may result in a shock and can damage components in the power supply.

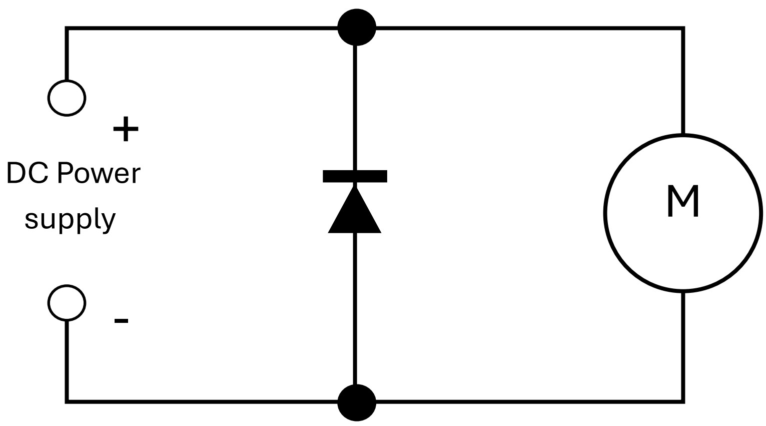

Figure 31 Placing a reverse-biased diode across a motor.

Since the magnetic field is provided by an electromagnet, simply reversing the current will not make the motor operate in reverse. To do this, we need to have a switch to change the polarity of the electromagnet and hence the direction of the magnetic field.

10.023 The Simple Electric Motor

This motor works on **direct current**. You may well have made something similar in class (*Figure 32*). They are often referred to as "**Westminster Motor Kits**".

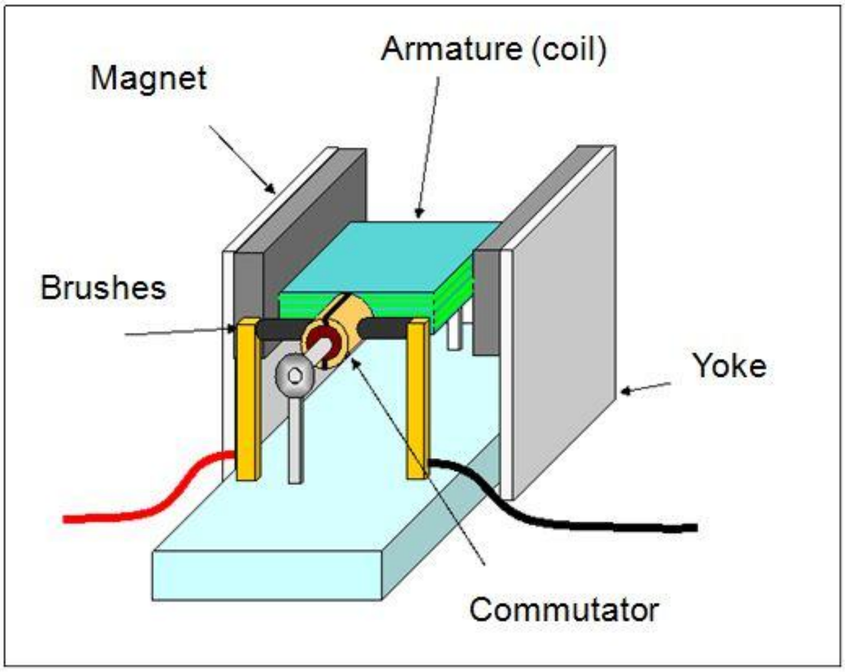

Figure 32 A simple DC motor

The current (*Figure 33*) is carried to the **armature** through carbon **brushes** and a split-ring **commutator**. The **commutator** acts as a change-over switch so that the current on the left hand side of the coil always moves towards us (and on the right hand side, away).

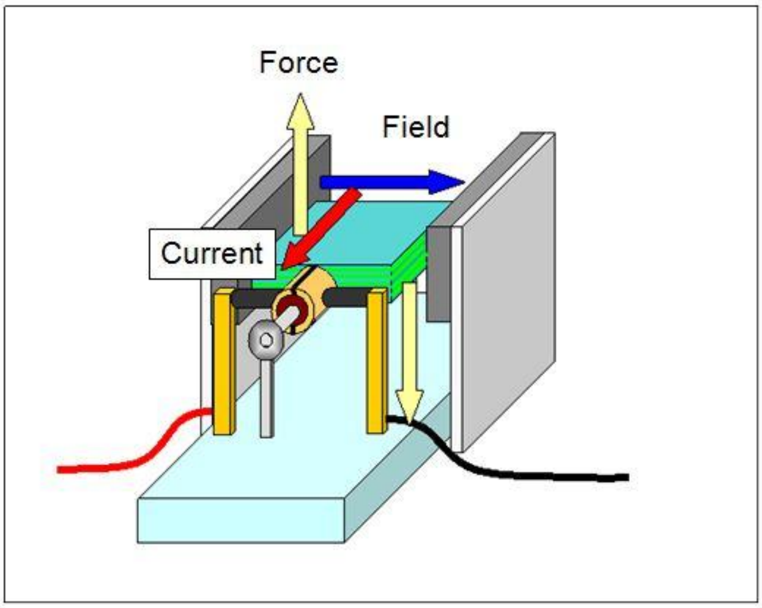

Figure 33 Applying Fleming's Left Hand Rule to the armature of a simple motor

The armature turns through 90 ° and the torque is zero (*Figure 34*).

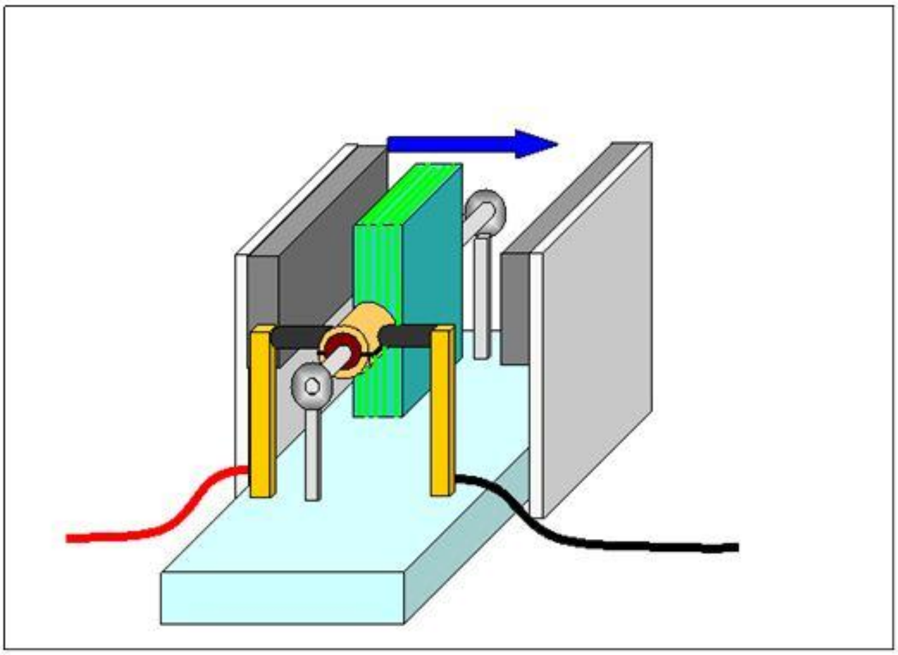

Figure 34 When the armature is at 90 degrees the torque is zero

In this case, we see that the armature is at 90 degrees, so that the current is at 0 degrees to the field.

There is zero torque, and the motor stalls.

To prevent this, small DC motors have 3 poles so that at least 1 pole is providing a force. The motor keeps on turning. Improved performance will be obtained with 5 or more poles. If we reverse the current, the direction of the rotation changes as well. In larger motors there are several coils. In larger motors, there are many pairs of poles. Only the poles that are connected to the brushes are providing a torque at any one time.

Note that the magnetic field is supplied by **permanent magnets**, so the field direction is always the same. If the current is reversed, the motor will rotate in the opposite direction.

10.024 Power of a motor

The power of the motor can be easily worked out. From A-level linear dynamics we learned that:

Power (W) = force (N) × speed (m s^{-1})

In Physics code:

$$P = Fv$$ …………… *Equation 15*

As a motor rotates, many of the concepts of rotational dynamics apply. In this case:

Power (W) = torque (N m) × angular velocity (rad s^{-1})

In Physics code:

$$P = \tau\omega$$ ....*Equation 16*

This equation is not on the AQA syllabus, but it may appear in a question, in which case it will be given.

11.025 AC Motor

In this simple **alternating current** motor, there is a permanent magnet that turns between two coils (*Figure 35*).

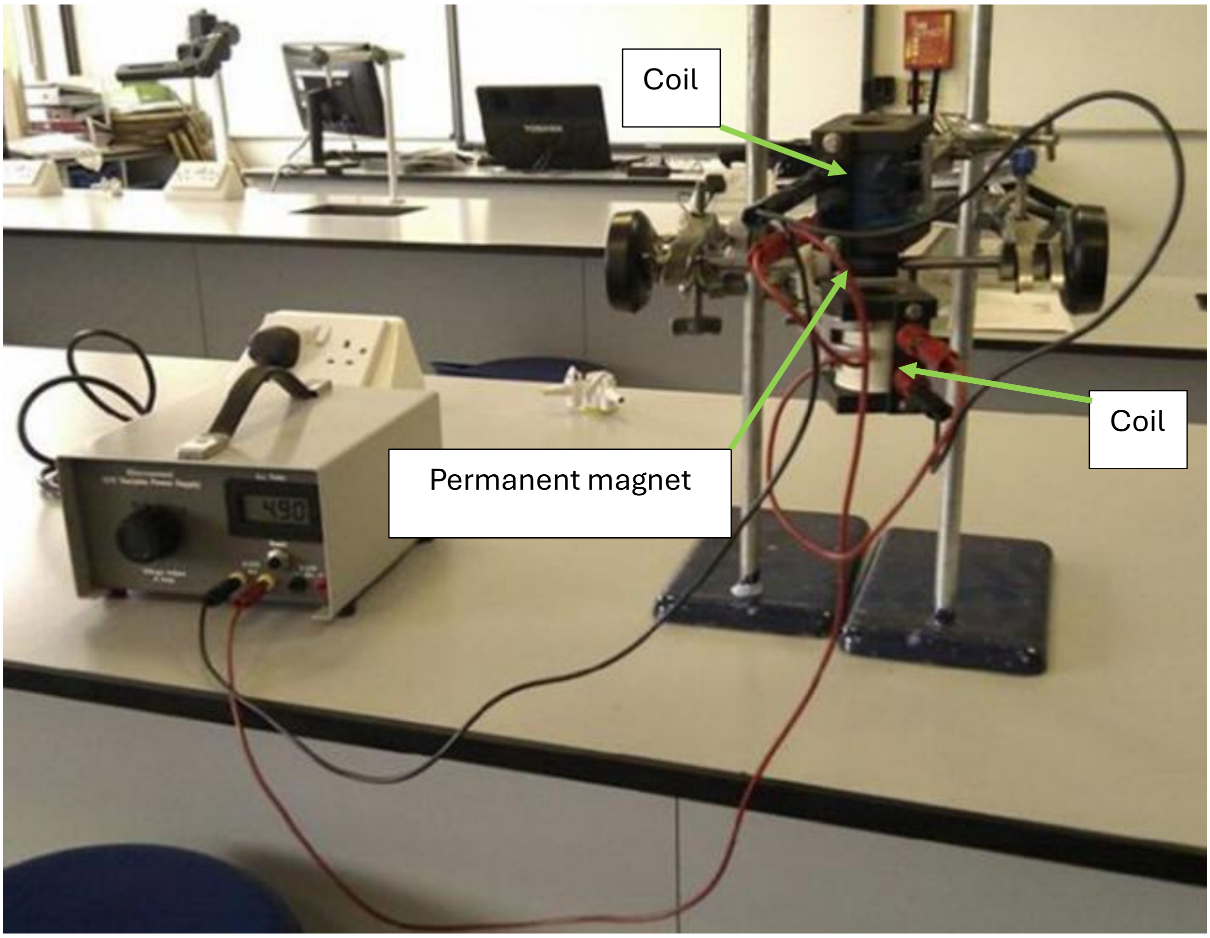

Figure 35 A very simple AC motor

This motor has a cylindrical magnet (from an old pond pump) which is mounted on a shaft held by clamps mounted on clamp-stands. There are coils connected in parallel, connected to an a.c. supply. Changing the voltage does not affect the speed of the motor.

In most a.c. motors there is no permanent magnet. Instead, there is a magnetic field induced in the **rotor** by the alternating current. Such a motor is called an **induction motor**. It does not work on direct current. The picture below (*Figure 36*) shows a small induction motor on a bench drill.

Figure 36 Small AC induction motor

Tutorial 11.02 Questions

11.02.1

A light coil of dimensions 3.0 × 5.0 cm with 500 turns is placed in a magnetic field of flux density 14 mT. A steady current of 0.25 A flows through it. Calculate:
(a) The maximum torque;
(b) The torque when the coil has turned through 27°.

11.02.2

A coil, which is part of an electric motor armature, has dimensions 4.5 × 6.0 cm with 500 turns. It is placed in a magnetic field of flux density 140 mT.

A steady current of 2.0 A flows through it. Calculate:

(a) The maximum torque;
(b) The power if the motor is spinning at 3000 revolutions per minute.

(c) The voltage of the source, if the motor is 80 % efficient.

Tutorial 11.03 Force on a Charge

All Syllabi

Contents

11.031 Interaction of Charged Particles with a Magnetic Field

We know that a magnetic field and an electric current interact to produce a force. Since a current is a flow of charge, it is reasonable to suppose that a magnetic field exerts a force on individual charge carriers.

We find that in a magnetic field, the force acts on a stream of electrons always at **90°** to the direction of the movement. Therefore, the path is **circular** (*Figure 37*).

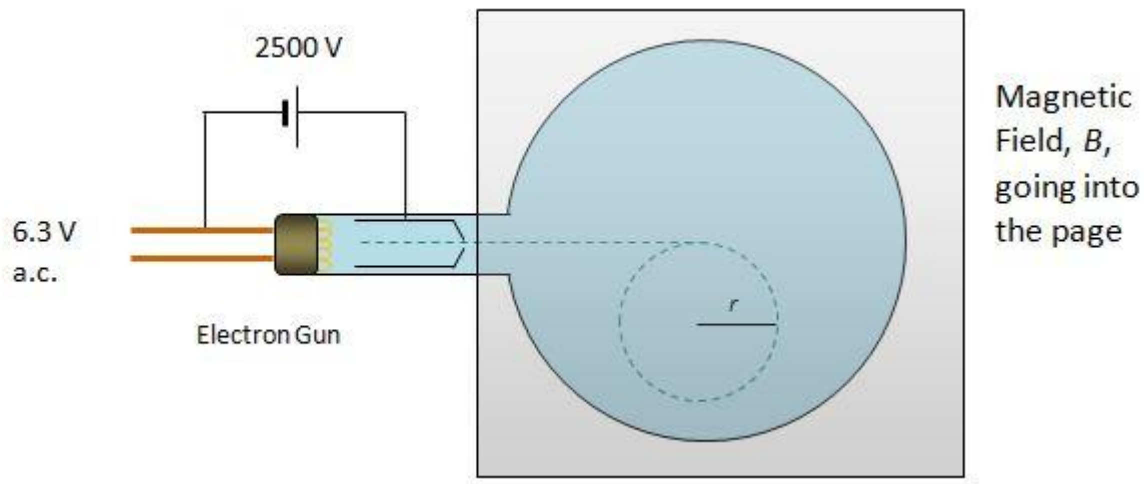

Figure 37 The path of charged particles in a magnetic field is circular

Consider a charge q moving through a magnetic field B at a constant velocity v. The charge forms a current that moves a certain distance, l, in a time t.

We know:

$$\text{Velocity} = \text{distance} \div \text{time} \quad \text{……… Equation 17}$$

$$\text{Current} = \text{charge} \div \text{time} \quad \text{……… Equation 18}$$

$$F = BIl \quad \text{……………… Equation 19}$$

We can substitute *Equations 17* and *18* into *19* to give us:

$$F = B \times (q/t) \times vt = Bqv \quad \text{……………… Equation 20}$$

So, the formula now becomes:

$$F = Bqv \quad \text{.................. Equation 21}$$

[F - force in N; B – field strength in T; q – charge in C; v – speed in m s^{-1}]

The charge is usually the **electronic charge**, 1.6×10^{-19} C.

If the magnetic field is at an angle θ to the magnetic field, the equation is modified to:

$$F = Bqv \sin \theta \quad \text{.............. Equation 22}$$

If no angle is mentioned in the question, assume that the angle is 90°.

Remember that the direction of the electrons' movement is in the **opposite** direction to the **conventional** current. So, if the electrons are going from left to right, the conventional current is going from right to left.

When using Fleming's Left Hand Rule, the current is **conventional**.

11.032 Path of charged particles in a Magnetic Field

We have seen that the force always acts on the wire at 90°, and that gives us the condition for **circular motion**.

We can combine the relationship

$$a = v^2/r$$ ……………… *Equation 23*

with Newton II to give us:

$$F = \frac{mv^2}{r}$$ ……………………*Equation 24*

Therefore:

$$Bqv = \frac{mv^2}{r}$$ ………………… *Equation 25*

The v on the left cancels to get rid of the v^2 term on the right:

$$Bq = \frac{mv}{r}$$ ………….. *Equation 26*

This rearranges to give us:

$$v = \frac{BQr}{m}$$ ……………. *Equation 27*

Worked Example   An electron passes through a cathode ray tube with a velocity of 3.7×10^{7} m s^{-1}. It enters a magnetic field of flux density 0.47 mT at a right angle. What is the radius of curvature of the path in the magnetic field?
Answer   Combine $$F = Bqv$$ and $$F = mv^2/r$$ to give: $$r = \frac{mv}{Bq}$$
$$r = \frac{9.11 \times 10^{-31}\ \text{kg} \times 3.7 \times 10^{7}\ \text{m s}^{-1}}{0.47 \times 10^{-3}\ \text{T} \times 1.6 \times 10^{-19}\ \text{C}} = \mathbf{0.39\ m} = 39\ \text{cm}$$

-

11.033 The Hall Effect

This section is for students for the Welsh Board and Eduqas Syllabuses. Students studying other syllabuses are, of course, welcome.

The **Hall Effect** was first discovered in 1879 by an American Physicist, Edwin Herbert Hall (1855 - 1938). He found that when a current-carrying conductor or a semiconductor is placed in a magnetic field, a voltage occurs that is perpendicular to the flow of the current. This is called the Hall Voltage.

The Hall Effect can be observed in conductors, semi-conductors, ionised gases, and plasmas. We will be looking at the effect of magnetic fields in electrons in semi-conductors, because the nature of semi-conductors allow for a relatively high voltage which is easy to measure. It is also easily reproduced in a school or college Physics laboratory. So, let's have a look at what happens.

The separation of charges leads to a potential difference, or voltage that is called the Hall voltage (V_H). This voltage also causes an electric field, *E* which is uniform. The idea is shown in the picture below (*Figure 38*).

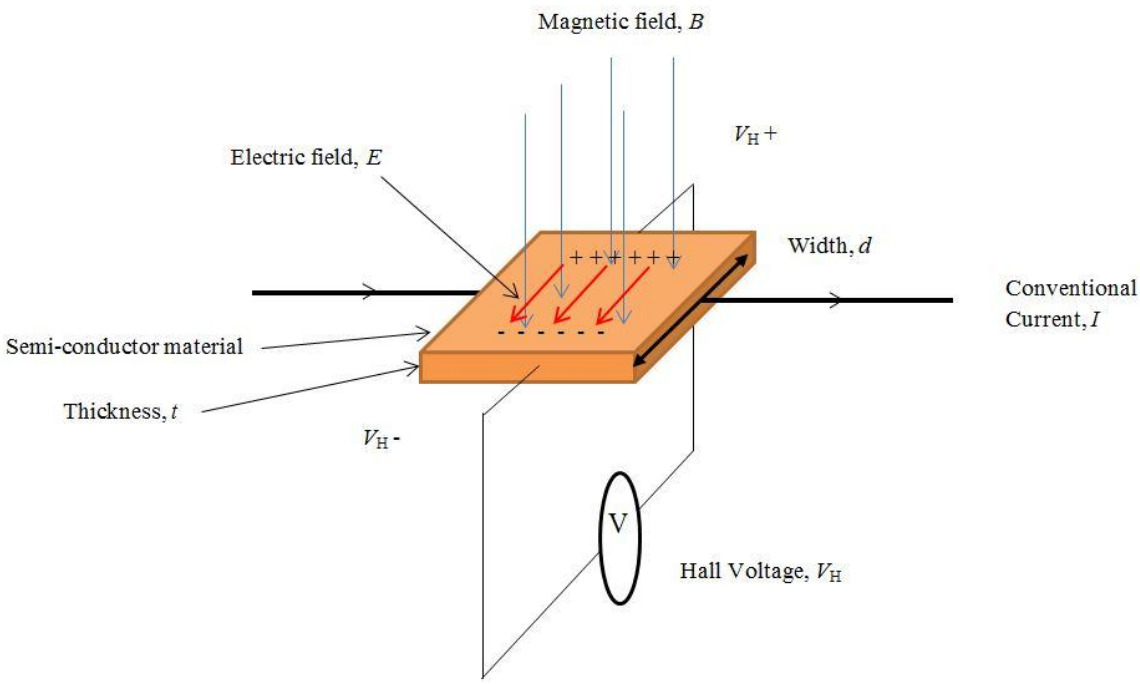

Figure 38 The Hall effect

We know that the electric field, E is given by:

$$E = \frac{V_{\mathrm{H}}}{d}$$

.... *Equation 28*

We also know from the definition of electric field that the force is:

$$F = Eq$$ ……………… *Equation 29*

So, we can write:

$$F = \frac{V_{\mathrm{H}} q}{d}$$

.... *Equation 30*

We also know that

$$F = Bqv$$

from *Equation 21*, so we can write:

$$Bqv = \frac{V_{\mathrm{H}} q}{d}$$

.... *Equation 31*

The q terms obligingly cancel out, and we can rearrange to give:

$$V_{\mathrm{H}} = Bvd$$

.... *Equation 32*

Measuring the speed of individual electrons is not at all easy, but we know that:

$$I = nAvq$$ …………… *Equation 33*

Where:

- n = number of charge carriers per unit volume (m^{-3}).
- A = area of the conductor (m^2).
- v = speed of charge flow ($m\ s^{-1}$).
- q = charge (C).
- I = current (A).

So, we rearrange to get:

$$v = \frac{I}{nAq}$$ .... *Equation 34*

and then substitute *Equation 34* into *32*, we get:

$$V_{\mathrm{H}} = \frac{BId}{nAq}$$ .... *Equation 35*

Now area, $A = dt$, so we can write:

$$V_{\mathrm{H}} = \frac{BId}{ndtq}$$ .... *Equation 36*

and the d terms cancel out to give us our final relationship:

$$V_{\mathrm{H}} = \frac{BI}{ntq}$$ ....*Equation 37*

where:

- V_H = Hall voltage.
- B = magnetic flux density (T).
- I = current (A).
- n = number of charge carriers per unit volume (m^{-3}).
- t = thickness of the slice (m).
- q = charge (C).

Note that n is the number of charge carriers per unit volume, not the number of turns per unit length.

The term dt in the argument above was distance × thickness to give area. It has nothing to do with a time interval!

The number of charge carriers per unit volume for typical semi-conductors is shown in the table:

Semi-conductor	n / m^{-3}
Gallium arsenide	1.10×10^{25}
Germanium	2.02×10^{21}
Pure Silicon	1.50×10^{16}

These figures are for a temperature of 300 K. The number of free charge carriers per unit volume rises as the temperature rises. We will use these, as college and university physics labs tend to have a temperature of about 300 K (27 °C). You would be well boiling if the temperature in the lab was 400 K.

For metals:

Metal	n / m-3
Aluminium	6.02×10^{28}
Copper	8.46×10^{28}
Tungsten	3.43×10^{28}

The Hall effect in theory can be observed in metals, but the Hall voltage would be so tiny that it's negligible. A calculation using the data above in a copper wire gives a Hall voltage of about 2×10^{-9} V.

<u>Uses of the Hall Effect</u>

In the school or college lab, the **Hall probe** is used to measure the value of a magnetic field. The picture shows two Hall probes (*Figure 39*).

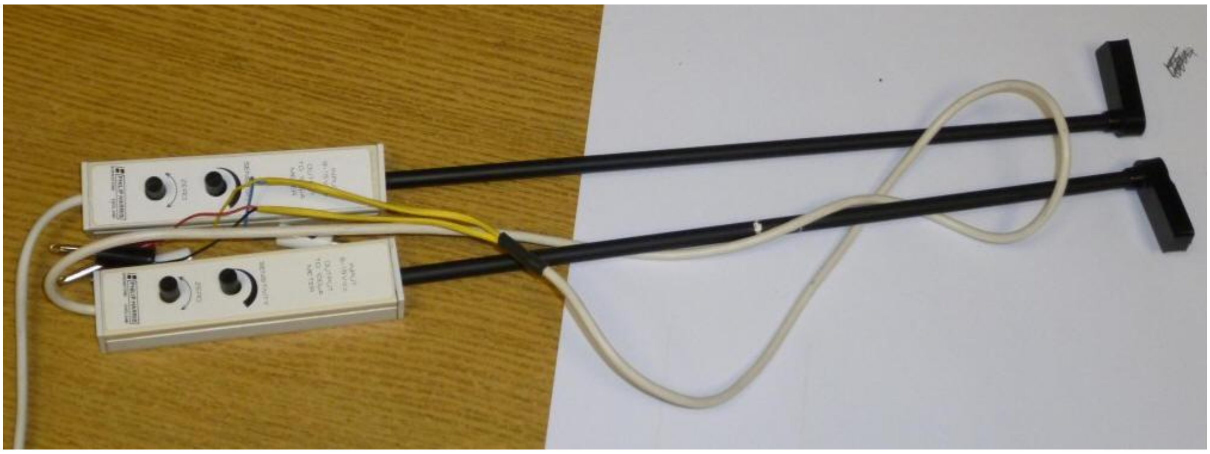

Figure 39 Hall probes

The Hall probes are **calibrated** using a magnetic field of known flux density to reduce uncertainty.

Hall probe sensors are used in motor control to detect the speed of the motor. Hall probe sensors are used to detect wheels locking in anti-lock braking systems.

11.034 Cyclotron

The **cyclotron** is a **particle accelerator** that relies on this idea. The machine's main components are two D-shaped electrodes ("Dees") in an evacuated chamber, placed between the poles of a large electromagnet (*Figure 40*).

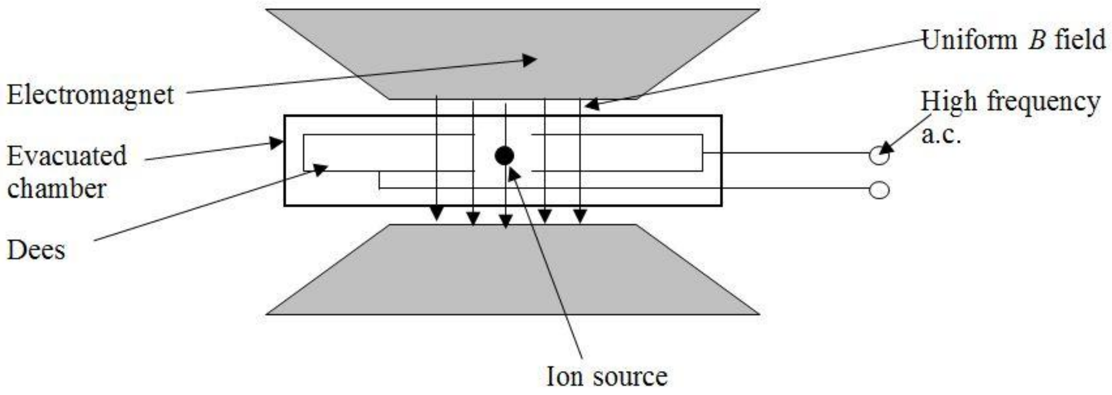

Figure 40 Side view of a cyclotron

From the top it looks like this (*Figure 41*).

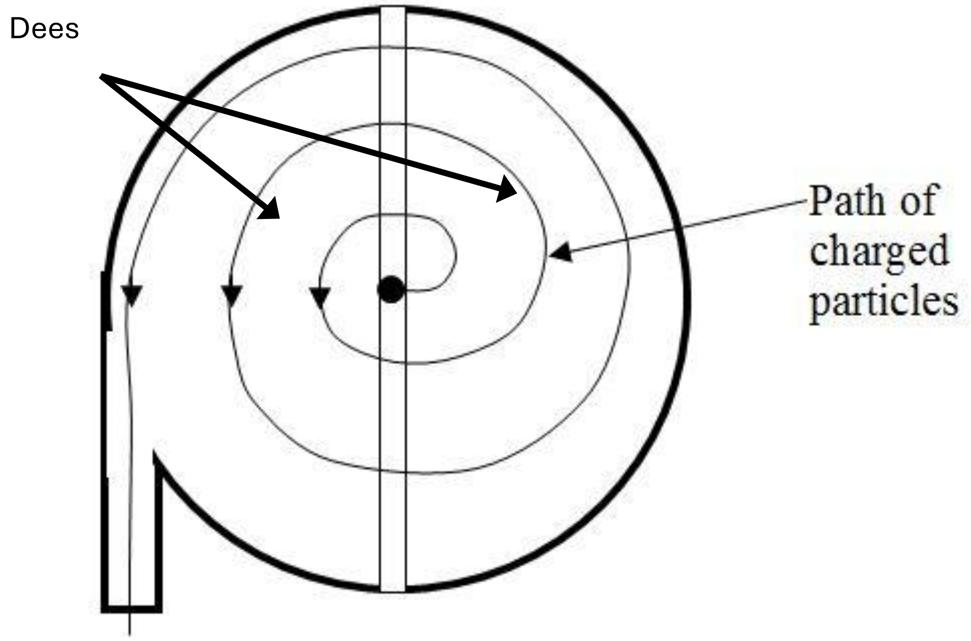

Figure 41 Cyclotron as seen from above

Notice that the beam of particles is not circular, but a **spiral**. This is because the particles are being accelerated by the electric field between each D-shaped electrode (called a **dee**). As their speed increases, so does the radius of the curved path.

If a particle of charge q enters one of the dees with a speed v, it will move in a semi-circular path of radius r.

$$Bqv = \frac{mv^2}{r}$$

.... *Equation 38*

Rearranging gives us

$$v = \frac{BQr}{m}$$

.... *Equation 39*

We can work out from

$$t = s/v$$ ……………… *Equation 40*

what time it takes for the charge to travel:

$$\frac{s}{t} = \frac{Bqr}{m}$$

.... *Equation 41*

For 1 revolution:

$$s = 2\pi r$$ ………………… *Equation 42*

Therefore:

$$\frac{2\pi r}{t} = \frac{Bqr}{m}$$

.... *Equation 43*

The *r* terms cancel:

$$\frac{2\pi}{t} = \frac{Bq}{m}$$

.... *Equation 44*

Since:

$$f = 1/t$$ …………….. *Equation 45*

we can replace the t term in Equation 44 to give:

$$2\pi f = \frac{Bq}{m}$$

.... *Equation 46*

Rearranging gives us:

$$f = \frac{Bq}{2\pi m}$$

.... *Equation 47*

11.035 Synchrotron

The largest particle accelerators are **synchrotrons**. The earliest machines had a **racetrack** arrangement as shown:

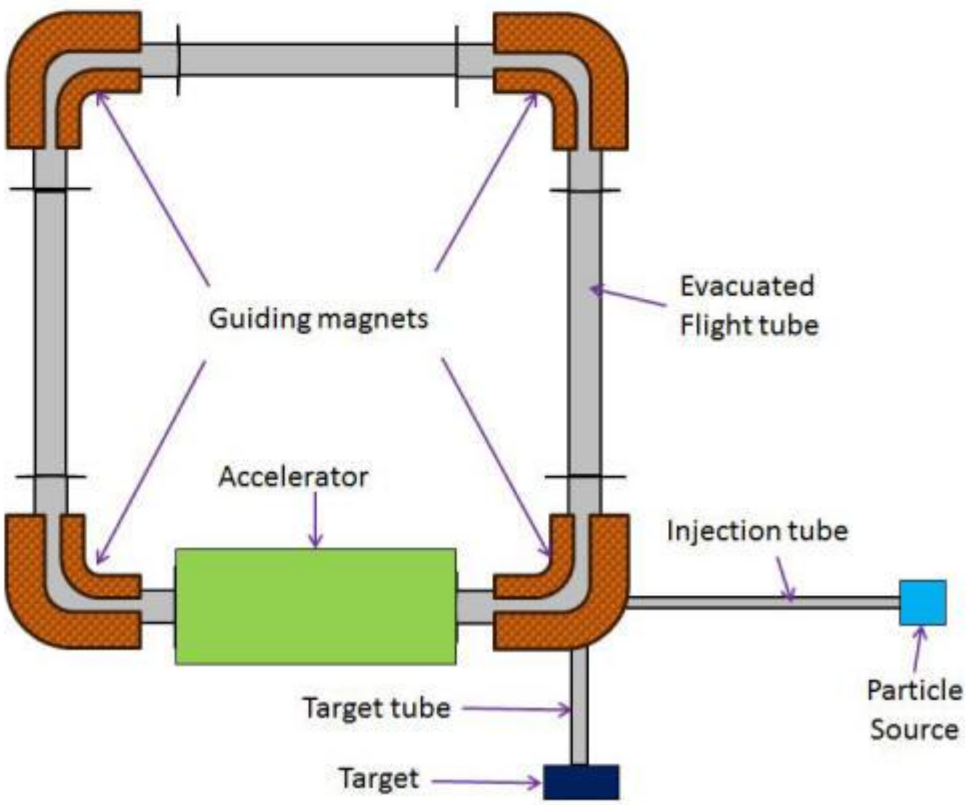

Figure 42 A synchrotron

The machine works like this:

- Charged particles are accelerated from the **particle source**. If they are **electrons**, they are produced by an **electron gun**. If the particles are **protons**, they are produced by **ionising** hydrogen atoms. Note that the particles must be **moving** before they enter the synchrotron. The synchrotron cannot accelerate a stationary particle.
- The particles from the source fly down the **injection tube**.
- The particles enter the synchrotron into the **flight tube**, which has a **vacuum** in it to reduce collisions with other particles.
- The particles are **accelerated** by the accelerator in the same way as a linear accelerator.
- The particles are **deflected** in a circular path by the **guiding magnets**.

- As the particle speed increases, the time to race around the track decreases. Therefore, the frequency at which they pass the accelerator increases.
- Therefore, the acceleration **frequency** has to go up.
- When the correct speed has been reached, the magnetic field in the guide magnet at the bottom right is reduced.
- Therefore, the particles fly down the **target tube** to hit the **target**. The target tube may also lead to another **accelerator**, or a **storage ring**.

The particles are in small groups rather than a continuous beam. The idea is that the particles get accelerated at the same point as they pass that point. For example, the electrons are going around the track 15 000 times a second, the accelerator must give the little brutes a kick up the backside 15 000 times a second. So, the acceleration is **synchronised** with the passage of the particles, hence the name synchrotron.

A synchrotron that accelerates electrons is called an **electron synchrotron**, and (what a surprise) a synchrotron that accelerates protons is called a **proton synchrotron**.

More modern machines are circular and have an arrangement like this:

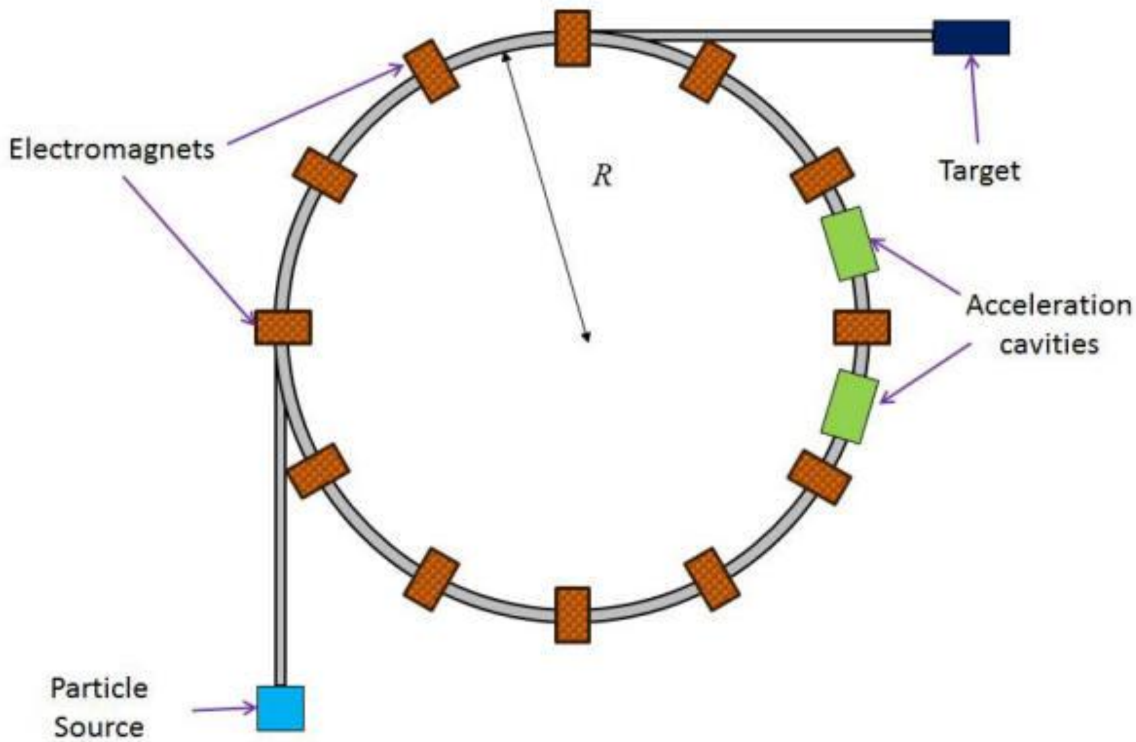

Figure 43 A more modern circular synchrotron

The particles may come from an electron gun, or a proton source, or may have been accelerated by a linear accelerator or another synchrotron.

Although the diagram shows the flight tube as **circular**, some machines have a **polygonal** flight tube with the electromagnets being rectangular. The path of the particles is **circular**. There may be several flight tubes to different targets which leave the ring **tangentially** as shown above. An extraction electromagnet may also be used to remove the particles from the ring.

A storage ring is similar, although there are no acceleration cavities (or they are turned off). The particles travel at the same speed in a storage ring and are extracted magnetically.

As the particles accelerate faster, **relativistic effects** become important, as the particles gain mass as they approach the speed of light.

Tutorial 11.03 Questions

11.03.1

An electron accelerated to 6.0×10^6 m s^{-1} is deflected by a perpendicular magnetic field of strength 0.82 T. What is the force acting on the electron? Would it be any different for a proton?

11.03.2

In a particle physics experiment, a detector is placed in a magnetic field of 0.920 T. A particle is found to produce a circular track of radius 0.500 m. Other experiments have shown that the particle carries a charge of $+1.60 \times 10^{-19}$ C and that its speed was 3.00×10^7 m s^{-1}.

What is the mass of the particle?

How does it compare to the mass of an electron (9.11×10^{-31} kg)?

11.03.3

The Hall voltage set up in a semi-conductor by a current in a magnetic field is 0.35 V. The semi-conductor slice is 5.0 mm wide.

Calculate the electric field strength.

11.03.4

A slice of germanium is 5.5 mm long, 3.3 mm wide, and 0.35 mm thick. It is carrying a current of 0.067 A and is placed in a magnetic field of flux density 0.14 T.

Calculate the Hall voltage if the charge on each charge carrier is 1.602×10^{-19} C.

11.03.5

A cyclotron has magnets of flux density 1.50 T and the polarity changes with a frequency of 2.00 MHz. A large, charged particle which has a charge of $+2e$ is inserted into the machine and is accelerated. Calculate the mass of the particle.

Tutorial 11.04 Magnetic Flux Density, Flux, and Flux Linkage	
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11.041 Flux Density	11.042 Flux
11.043 Flux Linkage	

11.041 Flux Density

We have seen how B is called the **magnetic field strength**, or the **flux density**, and is measured in Tesla, T. The magnetic flux density can be thought of as the concentration of field lines. We can increase the force by increasing any of the terms within the equation:

$$F = BIl$$ ……………. *Equation 48*

If we coil up the wire, we increase its length within the magnetic field. Flux density B is a **vector**.

If we look at the magnetic field of a solenoid, we know that it is like a bar magnet (*Figure 44*).

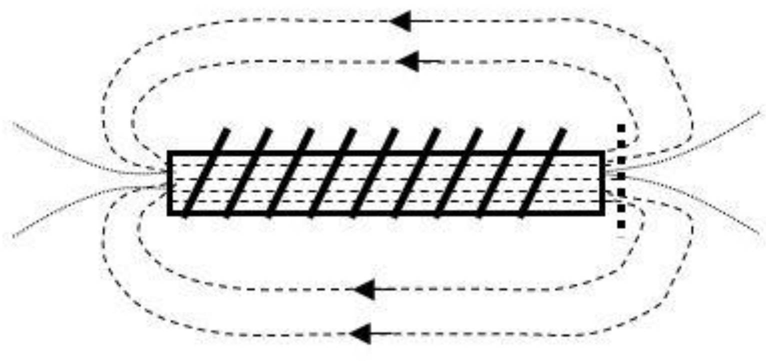

Figure 44 The magnetic field of a solenoid is like that of a bar magnet

We can see that the magnetic field strength is **uniform** within the solenoid. However, the flux density becomes less at the ends, as the field lines get spread out.

11.042 Flux

We need a term that tells us the **number of field lines**, and it is called the **magnetic flux**. It is given the physics code Φ ('Phi', a Greek capital letter 'Ph', or 'F'), and it has the units Weber (Wb), where:

$$1 \text{ Wb} = 1 \text{ T m}^2$$

or:

$$1 \text{ T} = 1 \text{ Wb m}^{-2}$$

The formal definition is:

The product between the magnetic flux density and the area when the field is at right angles to the area.

In code we write:

$$\Phi = BA \quad \text{………… } \textit{Equation 49}$$

Flux Φ is a vector.

Remember that flux density is the number of field line per unit **area**, not unit volume!

11.043 Flux Linkage

The **flux linkage** is the flux multiplied by the number of turns of wire. If each turn cuts (or links) flux Φ, the total flux linkage for N turns must be $N\Phi$. We can also write this as NBA. In other words:

Flux linkage = number of turns of wire × magnetic field strength × area

$$N\Phi = BAN \quad \text{………………… Equation 50}$$

The diagram (*Figure* 45) shows the situation when the flux linkage is the greatest.

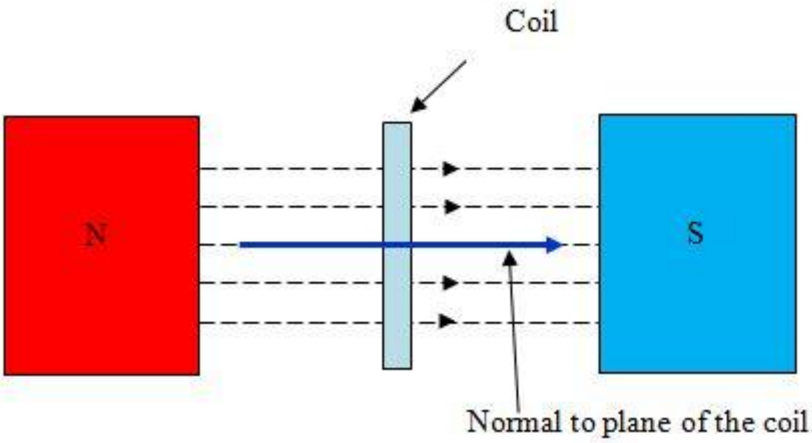

Figure 45 Flux linkage in a coil at 90 degrees to the magnetic field

Now we turn the coil through an angle θ (*Figure 46*).

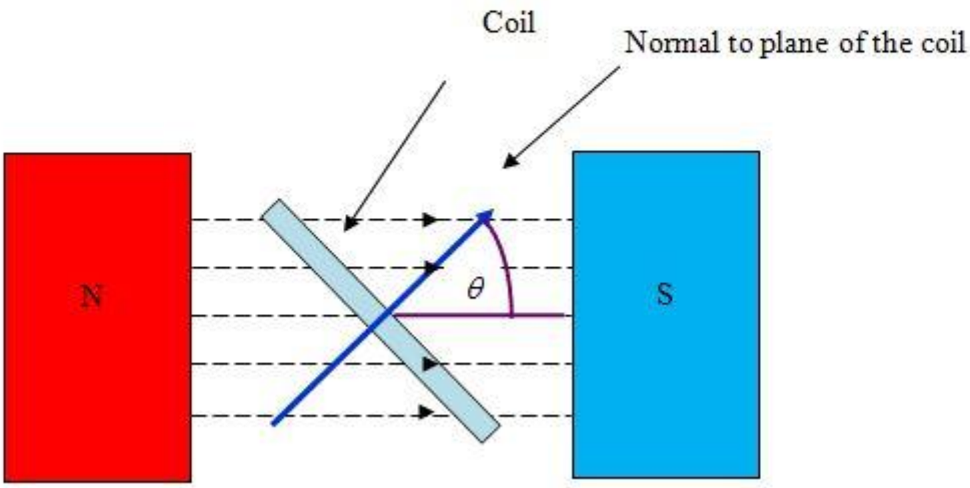

Figure 46 Turning the coil through an angle

We now have to change our formula to take the angle into account:

$$N\Phi = BAN \cos \theta$$ ……………… *Equation 51*

Where the flux linkage is the greatest, $\theta = 0$, hence $\cos \theta$ = 1. If the coil were parallel to the field, θ = 90° therefore $\cos \theta = 0$.

The flux linkage can be changed in two ways:

- We can alter the strength of the magnetic field.
- We can alter the area at 90° to the magnetic field by moving the coil. If we are turning the coil, the new flux linkage is given by $NBA \sin \theta$ where θ is **the angle the area makes to the magnetic field**. When we move a coil across a magnetic field, the area swept is the change in area (just like the change in distance is the distance moved).

We give the change in flux linkage the physics code $\Delta\Phi$.

Tutorial 11.04 Questions

11.04.1

How much flux links a 200 turn coil of area 0.1 m^2 when it is placed at 90^o to a magnetic field of strength 2.5×10^{-3} T?

11.04.2

The coil in Question 1 is now turned so that it makes an angle of $60\ ^o$ with the magnetic field lines. What is the change in flux linkage?

2. The Generator Effect

Tutorial 11.05 Electromagnetic Induction

All Syllabi

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11.05.1 Electromagnetic Induction

If we pass a current in a wire in a magnetic field, we know that the wire will move. It is therefore reasonable to suppose that if we move the wire in a magnetic field, and the wire is connected to an outside circuit, a voltage and current are **induced**. If the wire is not connected, a voltage only is induced. Consider this demonstration (*Figure 47*).

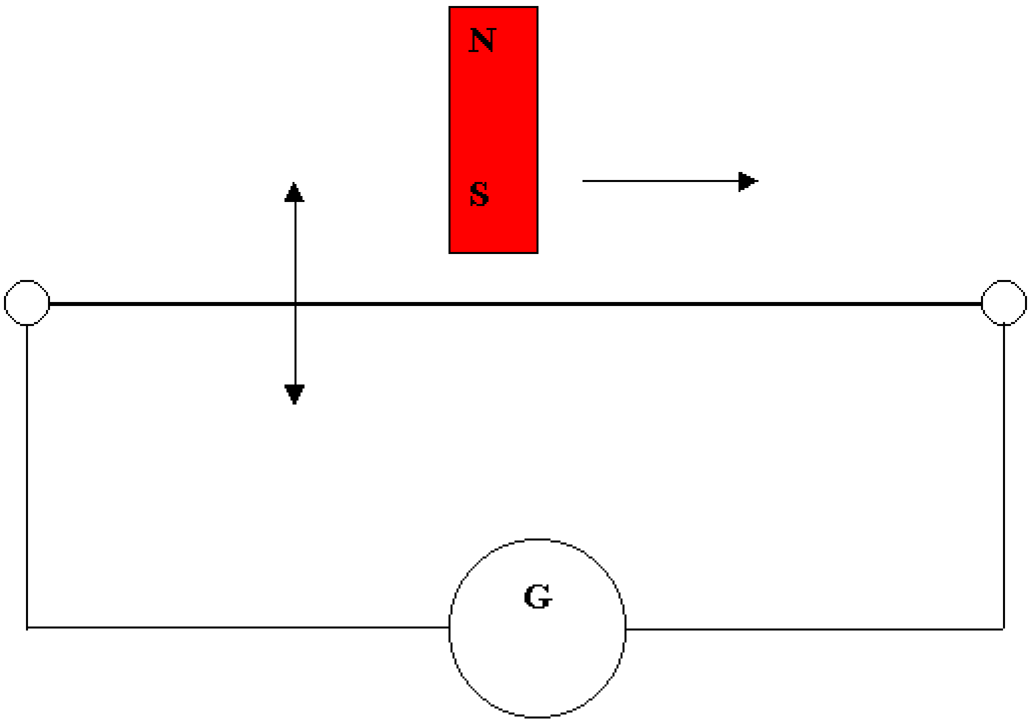

Figure 47 Inducing a current with a magnetic field

If we move the magnet parallel to the wire, the galvanometer hardly responds. However, if we move the magnet across the wire, then we see a definite reading on the galvanometer. The current (and voltage) induced on a single wire is rather small but is increased by having more turns of wire. For any voltage to be induced, we must move the magnet. We call this voltage the **induced electromotive force** (emf). It is often given the code $\mathcal{E}$, a fancy letter 'E'.

The emf depends on:

- The strength of the magnetic field.
- The speed at which you move the wire.
- How many turns of wire.

The maximum emf occurs when the wire is moved at 90 degrees to the field (*Figure 48*).

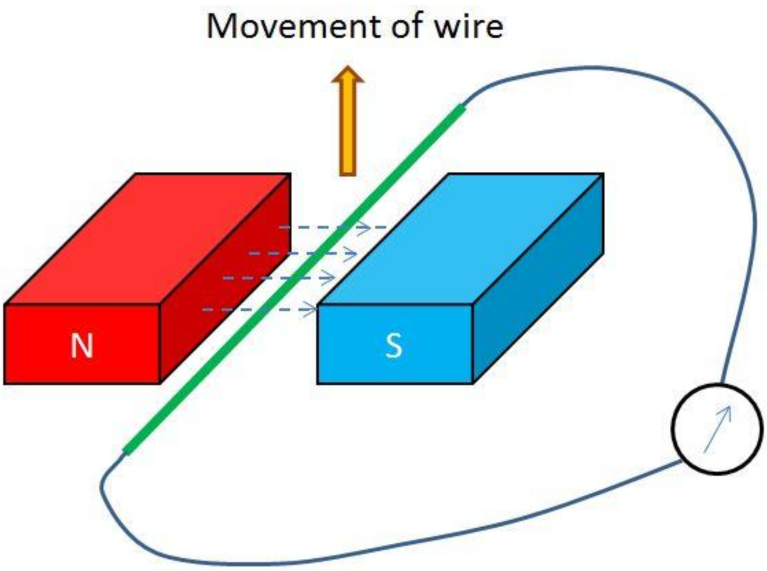

Figure 48 Moving a wire through a magnetic field

The direction of the current is determined by **Fleming's Right Hand Rule** (*Figure 49*).

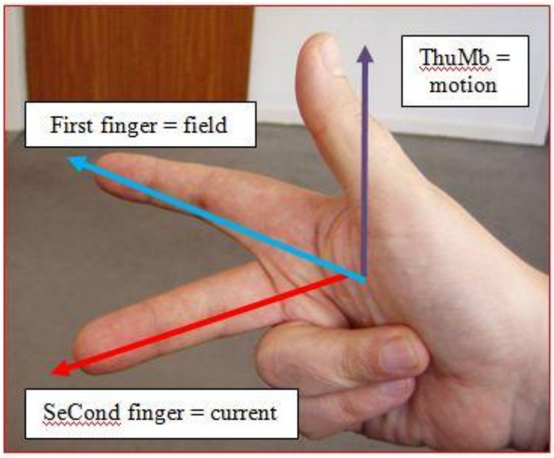

Figure 49 Fleming's Right Hand Rule

11.052 Faraday's and Lenz's Law

Faraday's Law and **Lenz's Law** are two important rules that govern this effect.

Faraday's Law is a formal definition of the effect:

The induced e.m.f. across a conductor is equal to the rate at which flux is cut.

Lenz's Law says:

The direction of any induced current is such as to oppose the flux change that caused it.

The induced e.m.f. sets up a current that would oppose the force that is pulling the wire. If the force were to assist the motion, we would get acceleration, and an increase in kinetic energy. This would break the Law of Conservation of Energy. In other words, we cannot get something for nothing.

Lenz's Law is important in motors and generators. As a motor speeds up, it acts like a generator to produce a **back e.m.f.** to oppose the current flowing in the motor. Therefore, the current through a fast running motor is quite small. When it is running slowly, a big current flows. Electric motors are therefore very suited to railway use, where big currents are needed to get trains moving, and there is no need for a gearbox that is needed for a diesel engine.

The effect that we have seen is summed up in the relationship:

$$\mathcal{E} = -N\left(\frac{\Delta\Phi}{\Delta t}\right)$$ .... *Equation 52*

[N - number of turns; $\mathcal{E}$– e.m.f. (V); $\Delta\Phi/\Delta t$ – rate of change in flux (Wb s^{-1})]

Worked Example A single turn of wire of cross-sectional area 5.0 cm^2 is at 90° to a magnetic field of 0.02 T, which is reduced to 0 in 10 s at a steady rate. What is the e.m.f. induced?
Two formulae to use: $$\Phi = BA$$ and $$\mathcal{E} = -N\left(\frac{\Delta\Phi}{\Delta t}\right)$$
We need to work out the flux: $\Phi = BA$ = 0.02 T × 5.0 × 10^{-4} m^2 = 1.0 ×10^{-5} Wb
Now we can work out the e.m.f: $$\mathcal{E} = -\frac{N\Delta\Phi}{\Delta t} = \frac{1 \times 1.0 \times 10^{-5}\text{ Wb}}{10\text{ s}} = \mathbf{1.0 \times 10^{-5}\ V}$$

11.053 Cutting Flux

Consider 2 coils cutting field lines in a magnetic field of flux density B. Both are moving at constant speed v (*Figure 50*).

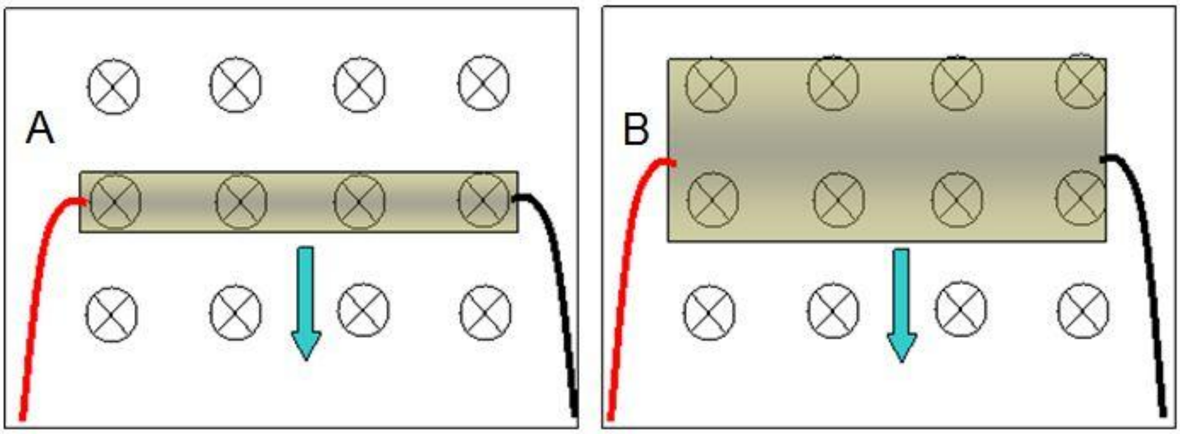

Figure 50 Cutting flux

In diagram A we can see that a certain number of field lines are being cut every second. In diagram B we see that the area is doubled, so twice as many field lines are cut every second.

Since $\Phi = BA$, there is twice as much flux cut every second.

11.054 Linking EMF and the speed of a wire in a magnetic field

Consider a wire on two rails in a magnetic field of strength B Tesla, w metres apart, travelling a distance l metres at a velocity of v metres per second in a time of t seconds (*Figure 51*).

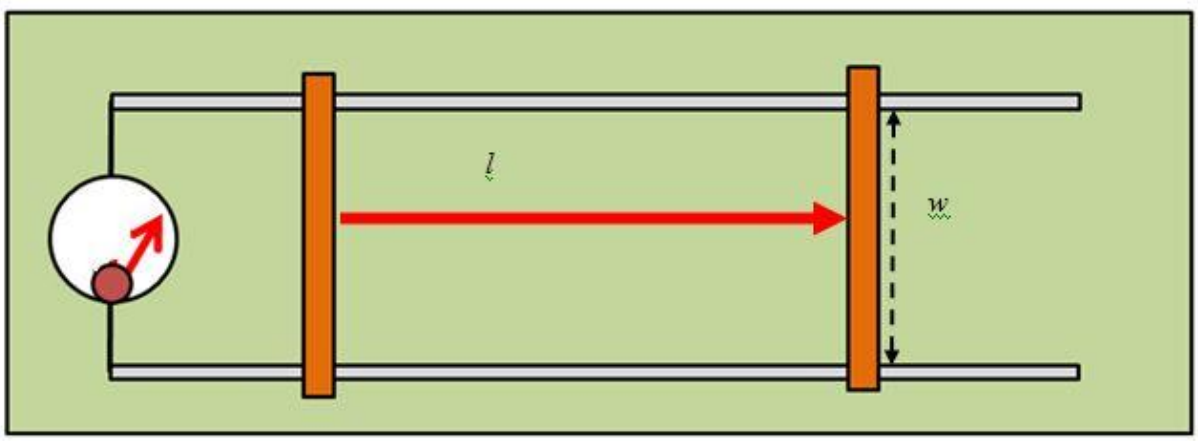

Figure 51 Moving a wire along fixed rails in a magnetic field

Faraday's and Lenz's Laws:

$$E = -N\frac{\Delta\Phi}{\Delta t}$$ .... *Equation 53*

The minus sign shows that Lenz's Law applies.

Since $\Phi = BA$, we can write:

$$E = -NA\frac{\Delta B}{\Delta t}$$ .... *Equation 54*

Also, $A = lw$ and $l = vt$. So, we can write:

$$E = -Nlw\frac{\Delta B}{\Delta t}$$

.... *Equation 55*

And then:

$$E = -Nvtw\frac{\Delta B}{\Delta t}$$

.... *Equation 56*

The t terms cancel out to give us:

$$E = (-)NvwB$$

.... *Equation 57*

The minus sign is there to satisfy Lenz's Law.

Questions on this often involve the rather fatuous example of aeroplanes flying through the Earth's magnetic field. An e.m.f. is induced due to the vertical component of the Earth's magnetic field. It's no damned good to the aeroplane, which would have to fly along fixed rails to generate anything useful – a sky-train?

11.055 Magnetic Fields in Coils

We can move a wire through a magnet to get an e.m.f., OR we can move a magnet through a coil of wire. It doesn't matter, as long as there is relative movement (*Figure 52*).

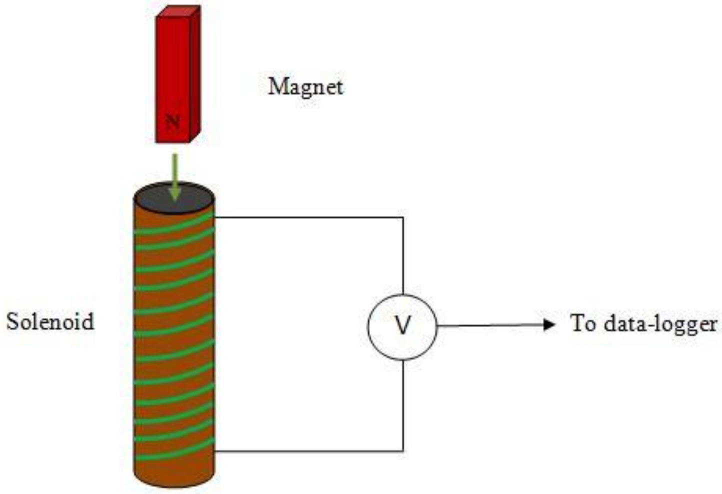

Figure 52 Moving a magnet through a solenoid

When the magnet is moved towards the **solenoid**, we get a voltage induced according to Faraday's Law. However, Lenz's Law tells us that the **direction** of the current in the solenoid will make a magnet that will **oppose** the movement of the bar magnet. In other words, a North pole is induced, and it will try to repel the magnet (*Figure 53*).

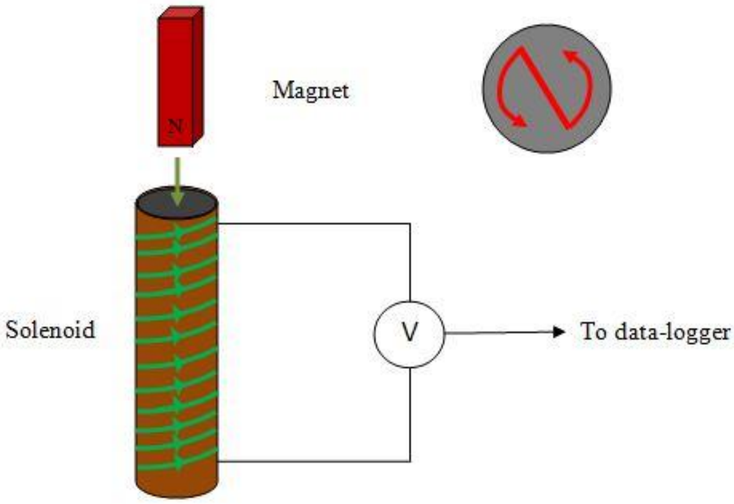

Figure 53 As a North pole moves towards a solenoid, a North pole is induced

The current goes **anticlockwise**. If you put arrows on the ends of the N (for North), you will see it going anticlockwise.

Now let's think about the magnet in the middle, as shown in the next diagram (*Figure 54*).

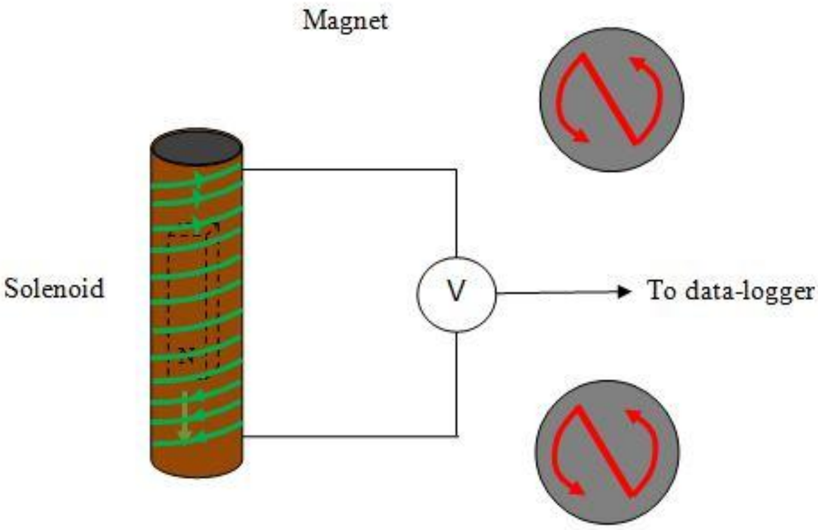

Figure 54 A magnet falling through the solenoid

In this instance we see three different things:

- At the top, we see a **North pole**. This is because the South pole of the magnet is moving downwards. Lenz's Law tells us that the direction of the current opposes the movement, so the current is trying to attract the magnet back to it.
- In the middle, the field lines are **parallel** to the direction of movement. There is no change in flux, so no e.m.f. is induced.
- At the bottom, a **North pole** is induced to repel the North pole of the magnet.

You can see that the induced voltages are going in the opposite direction, so there is **zero** overall e.m.f. at this point.

Now the magnet is coming out at the bottom (*Figure 55*).

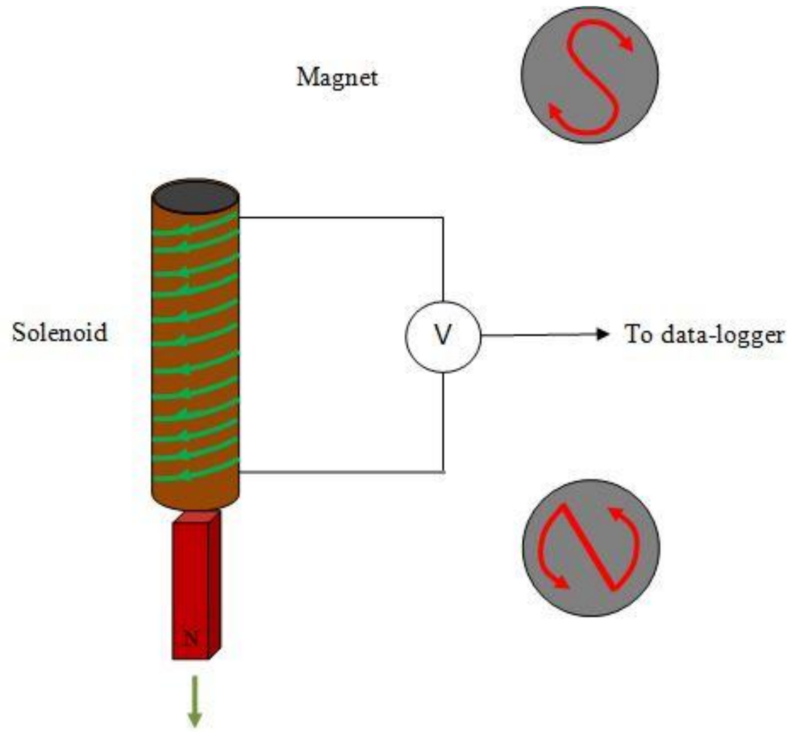

Figure 55 The magnet passes the bottom of the solenoid

In this case, the South pole of the magnet is inducing a **North pole** at the bottom of the coil, which is trying to attract it back. At the other end of the coil, there is a South pole induced.

The graph shown by the data-logger looks like this (*Figure 56*).

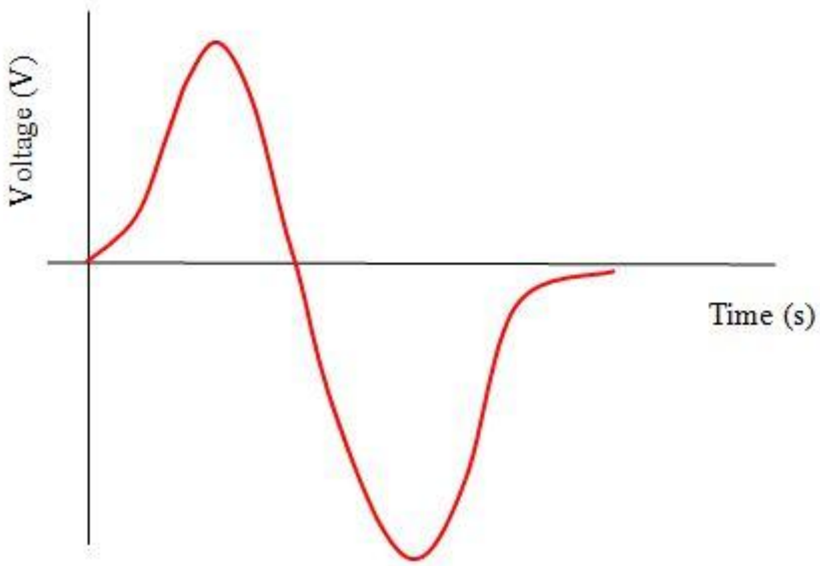

Figure 56 Induced voltage of a magnet falling through a solenoid

Notice that the second peak has a higher (negative) value. This is because the magnet is accelerating, so its downwards velocity is changing all the time.

If the coil is connected to a voltmeter, the acceleration of the magnet will be very close to 9.8 m s^{-2}, because the current will be very small, and the opposition to the movement will be tiny. However, if there is a low resistance in the external circuit, there will be a noticeable effect on the acceleration.

The area under the graph is the **change in flux**.

11.056 The Transformer Effect

We can use a magnetic field to induce a voltage in two ways:

1. **Relative movement**. The size of the voltage depends on:
   - Speed the magnet passes through a coil or vice versa.
   - Number of turns in the coil.
   - Strength of the magnet.
2. **Changing a magnetic field**. We don't have to make the magnetic field move. If we turn the current on or off, there is a change in the magnetic field, and that induces a voltage in a second **unconnected** coil. This is called the **transformer effect** or **mutual induction**.

We can also make the magnetic field go forward and backwards by using an alternating current. We will look at the transformer effect in a later tutorial. However, it is worth noting now that radio broadcasts use the transformer effect. The changing magnetic field (made by the alternating current) in the transmitter induces a very tiny alternating voltage in the receiver. This is amplified to enable us to hear the broadcast.

11.057 Investigation of the effect on magnetic flux linkage of varying the angle using a search coil and oscilloscope (Required Practical)

This experiment uses a circular coil which is connected to an AC supply and a search coil connected to a CRO. The apparatus is set up like this (*Figure 57*).

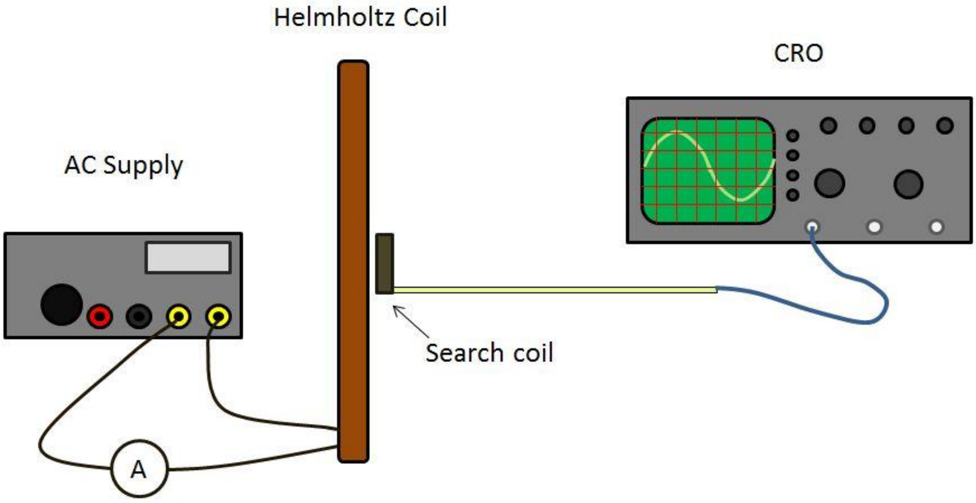

Figure 57 Apparatus for the required practical

The search coil is clamped so that it remains in the same place. It should be placed at the centre point of the flat coil (sometimes called a Helmholtz coil). The coil is then rotated to an angle like this and the display on the CRO will change (*Figure 58*).

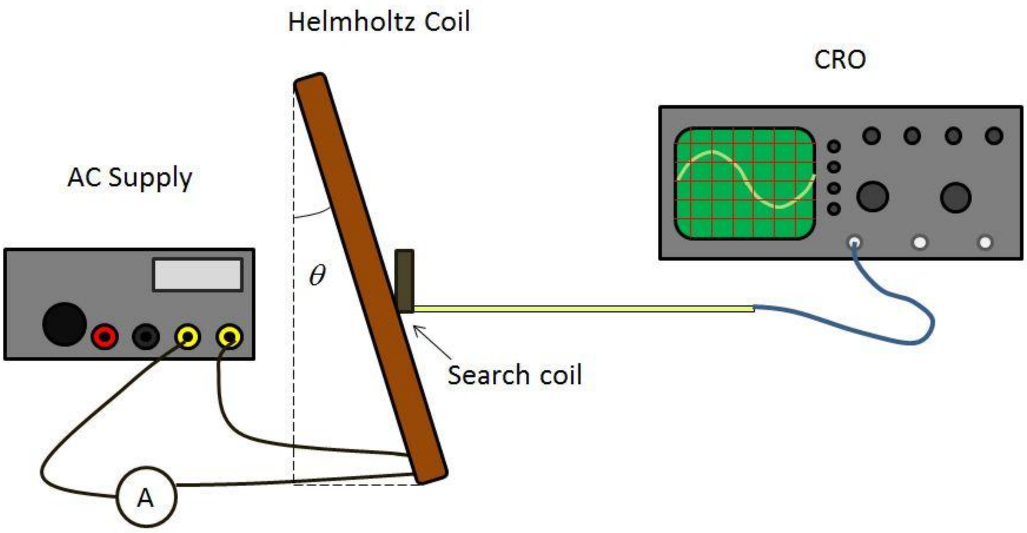

Figure 58 Changing the angle

The flat coil will need to be held in a clamp to make sure that the angle doesn't change. If your coils are like this (*Figure 59*), then they can be easily held with a boss on a clamp-stand.

Figure 59 Typical flat coils used in the required practical

You will need to do a preliminary experiment to determine the best current to use. The flat coils shown can carry a maximum of 2 amps, which should not be exceeded, otherwise they will get hot. They are expensive. An ammeter can be inserted to ensure that the current is not excessive. You will need an AC ammeter. If you have a power signal generator, you might want to determine the best frequency. You will also need to set up the CRO to give a reading that reduces the uncertainty to a minimum. You also should consider ways of measuring angles with less uncertainty than using a protractor. The diagram above might give you a few hints...

Do NOT be tempted to use an AC voltmeter in this assessment. Part of the assessment is how well you can use the CRO.

The experiment will NOT work with DC.

See Tutorial 11.08 on how to use the CRO.

You should include details of the preliminary experiment in your report, including your chosen values and ranges.

You will need harvest data from angles 0° to 90°. You need to measure the voltage on the CRO. You will need to take repeats as well.

The theory tells us that:

$$\mathcal{E} = -BANf\cos\theta$$

.... *Equation 58*

The graph of EMF against $\cos\theta$ should look like this (*Figure 60*).

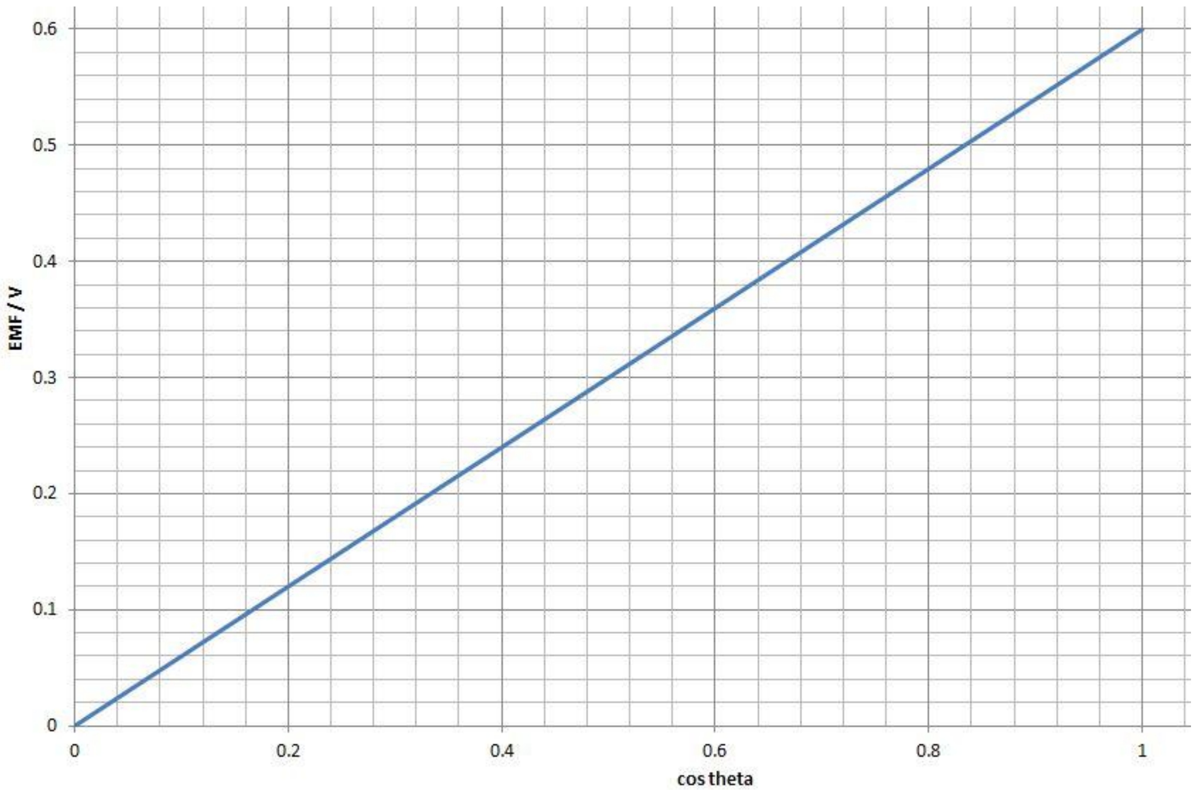

Figure 60 Graph of e.m.f. against cos θ

The gradient of the graph can be measured easily:

$$\text{Gradient} = BANf$$

……………. *Equation 59*

Therefore:

$$B = \frac{\text{Gradient}}{ANf}$$

.... *Equation 60*

You could also compare the flux density worked out from the gradient with the flux density produced by the coil. Hopefully they should be about the same...

11.058 Theory

We know from Faraday's and Lenz's Law that:

$$E = -N\frac{\Delta\Phi}{\Delta t}$$

.... *Equation 61*

We know also from our previous tutorial that:

$$N\Phi = BAN\cos\theta$$ .... *Equation 62*

We can combine these to write:

$$\mathcal{E} = -\frac{N\Delta\Phi}{\Delta t} = -\frac{\Delta BAN\cos\theta}{\Delta t}$$

.... *Equation 63*

Getting rid of the Δ term, we can write:

$$\mathcal{E} = -\frac{BAN\cos\theta}{t}$$

.... *Equation 64*

Since $f = 1/t$, we can now write:

$$\mathcal{E} = -BANf \cos\theta$$

.... *Equation 65*

The minus sign shows that there is a phase change between the current in the coil and the emf induced in the search coil.

We can measure $\mathcal{E}$ on the CRO. N is the number of turns which will be written on the search coil. Area, A, is easily worked out by measuring the diameter of the search coil. The frequency, f, can be checked on the CRO as well. It should be about the same as the frequency displayed on the signal generator, depending on how well the signal generator is calibrated. If you are using an AC supply, the frequency is 50 Hz. The problem comes with measuring B, the flux density.

Consider a flat coil of N turns, and radius r metres. It is carrying a current of I amps. The current is going clockwise as we look at it (*Figure 61*).

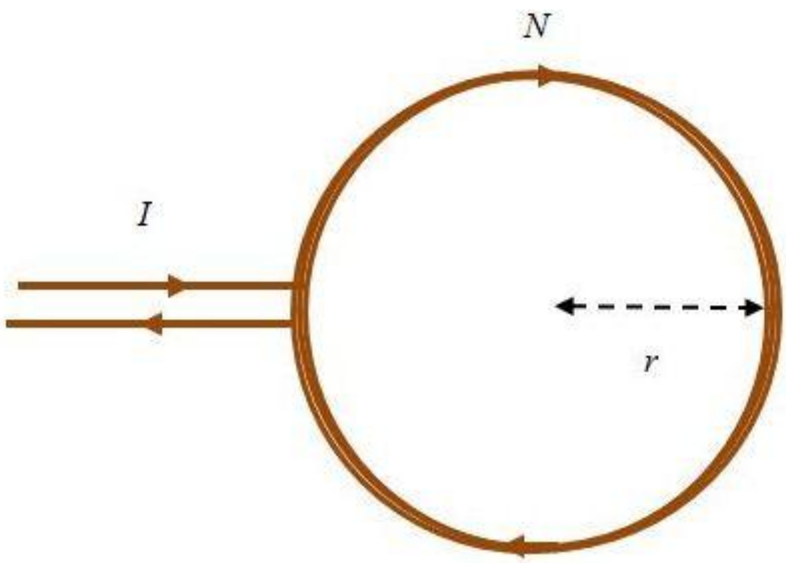

Figure 61 Current flowing through a flat coil

The magnetic field looks like this (*Figure 62*).

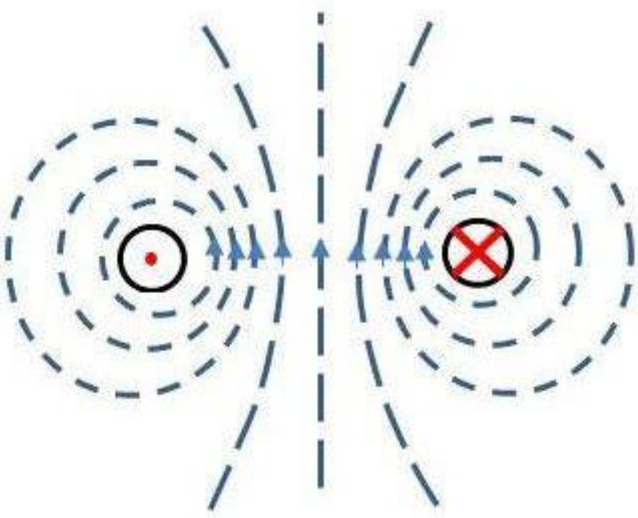

Figure 62 Magnetic field produced by a flat coil

The maximum magnetic field strength is at the very **centre** of the coil. The flux density at the centre is directly proportional to the current, I and the number of turns, N. It is inversely proportional to the radius, r, of the coil.

The formula for the flux density at the centre is this:

$$B = \mu_0 \frac{NI}{2r}$$

.... *Equation 66*

The term μ_0 ("mu-nought") is a constant which has the value $4\pi \times 10^{-7}$ H m^{-1} (Henry per metre). It is called the **permeability** of free space.

Do not mix up the permeability of free space μ_0 with the permittivity of free space ε_0. The two words sound similar.

The number of turns for the flat coil will be different to the number of turns in the search coil. The radii will be different as well.

Tutorial 11.05 Questions

11.05.1

A search coil has 2500 turns and an area of 1.5×10^{-4} m^2. It is placed between the poles of a large horseshoe magnet. It is rapidly pulled out of the field in a time of 0.30 s. A data-logger records an average value for the emf of 0.75 V. What is the flux density between the poles of the magnet?

11.05.2

A coil of length 50 mm and width 80 mm with 30 turns is passed through a perpendicular magnetic field of value 0.245 T at a velocity of 1.20 m s^{-1}.

(a) Which data item is irrelevant?

(b) Calculate the EMF.

11.05.3

A conducting liquid flowing through a pipe in a magnetic field cuts lines of magnetic flux and generated an emf across opposite sides of the liquid. The emf can be used to determine the flow rate of the liquid.

In a brewery beer flows through a 35 cm diameter pipe at a rate of 0.4 m^3 s^{-1}. The pipe is in a magnetic field of 5.0×10^{-3} T.

What is the emf between the opposite sides of the liquid?

Tutorial 11.06 Alternating Currents	
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11.061 What is AC?

Direct current from a battery moves in one direction only, from positive to negative. In **alternating current,** the direction is changing all the time. The charge carriers are moving forwards and backwards many times a second. In Europe it is 50 Hz (cycles per second); in the USA 60 Hz.

AC and DC are equally good at heating, lighting, or running motors. DC is essential for chemical processes such as electrolysis. Low voltage DC is used in electronic devices.

AC is much more easily distributed than DC. This is because transformers use AC only; they cannot work with DC. So, electricity is distributed at very high voltages (275 kV) at relatively low currents. As a result, only a small proportion of the transmitted energy is lost as heat in the wires. The picture (*Figure 63*) shows a transformer at a power station.

Figure 63 Transformer at a power station

The symbol for an alternating supply is shown below (*Figure 64*).

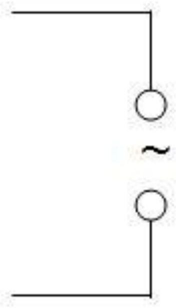


Figure 64 Symbol for AC supply

The graph (*Figure 65*) below shows the difference between AC and DC.

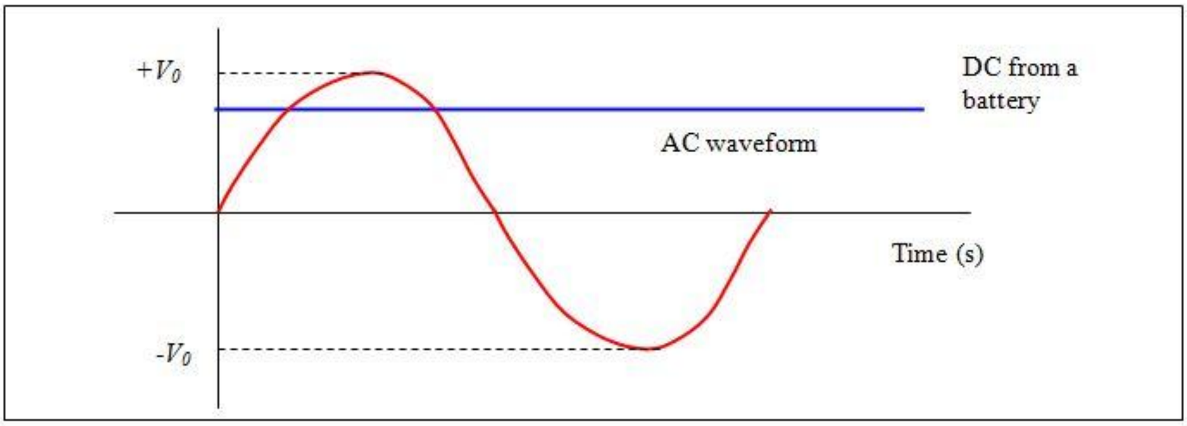


Figure 65 Comparing AC and DC

- One complete alternation is called a **cycle** (NOT wavelength). The **frequency** is the number of cycles per second. Units are **hertz** (Hz).
- The **period** is the time taken for one cycle. It is measured in seconds. $f = 1/T$.
- The current follows exactly the same wave form as voltage.
- The graph is called a **sinusoidal waveform** or a **sine wave**.

These features are shown on the graph (*Figure 66*).

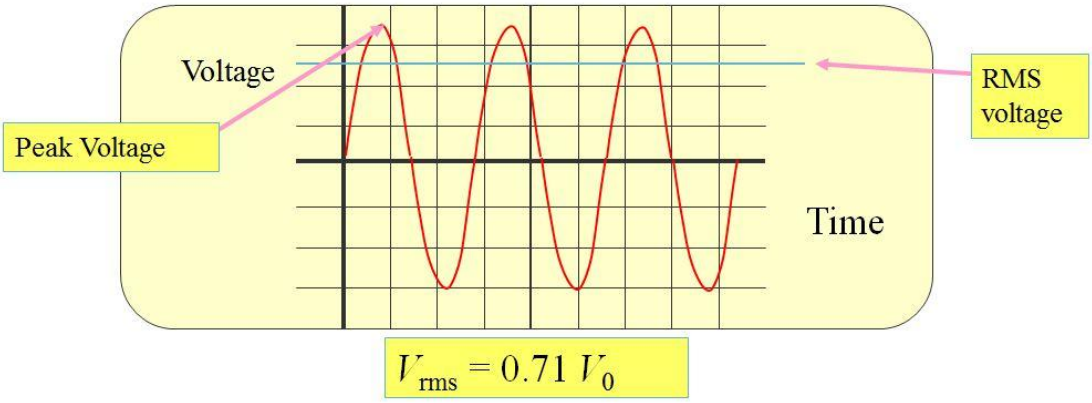


Figure 66 Feature of an AC waveform

On the CRO the AC waveform looks like this (*Figure 67*).

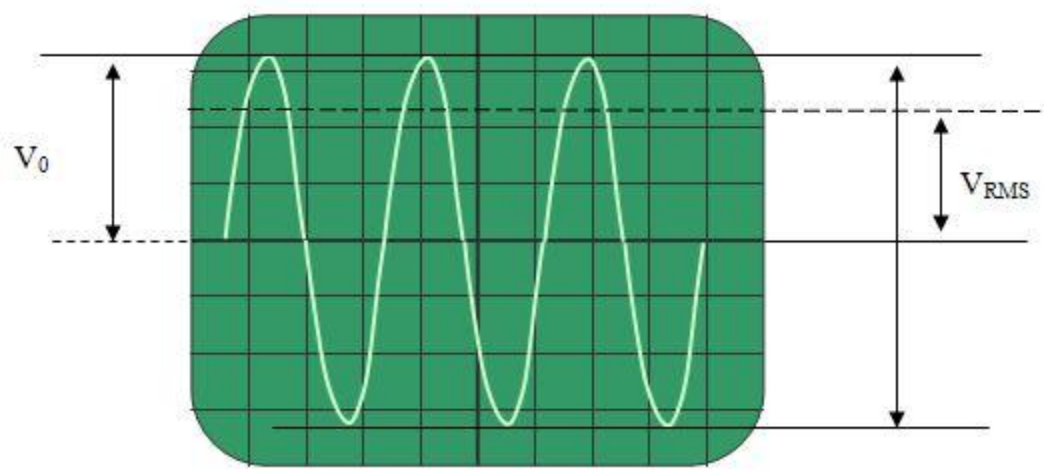


Figure 67 AC waveform on a CRO screen

Note the **peak voltage** is V_0 (sometimes called V_{pk}). Notice also the **effective voltage**, V_{rms}. On a CRO, there is less uncertainty if you measure from the peak to the trough. This is called the **peak-to-peak voltage**, and is given the code $V_{pk\ to\ pk}$.

The peak to peak voltage is **twice** the peak voltage. Remember to halve this when you do calculations with the peak voltage.

11.062 Root Mean Square Value

The values of voltage and current are constantly changing in AC, unlike in DC in which they are steady. We can measure AC voltages in two ways:

- Measure the **peak to peak** voltage, easily done on a cathode ray oscilloscope (CRO).
- Measure the **root mean square** (rms) value, or the **effective** value.

We use the **rms value** because its use allows us to do electrical calculations as **if they were direct currents**. Root mean square values of voltage and current are related to the peak values by the following simple equations:

$$I_{rms} = \frac{I_0}{\sqrt{2}} \text{ .... } \textit{Equation 67}$$

and

$$V_{rms} = \frac{V_0}{\sqrt{2}} \text{ .... } \textit{Equation 68}$$

11.063 How Does the Power Vary?

We know that:

$$\text{Power} = \text{volts} \times \text{amps} = V_{rms} \times I_{rms} \text{ ……………… } \textit{Equation 69}$$

so:

$$\text{Peak power (positive)} = (V_{rms} \times \sqrt{2}) \times (I_{rms} \times \sqrt{2}) = 2P \text{ ………… } \textit{Equation 70}$$

Minimum power = 0

$$\text{Peak power (negative)} = (-V_{rms} \times \sqrt{2}) \times (-I_{rms} \times \sqrt{2}) = +2P \text{ …………… } \textit{Equation 71}$$

The graph shows the idea (*Figure 68*).

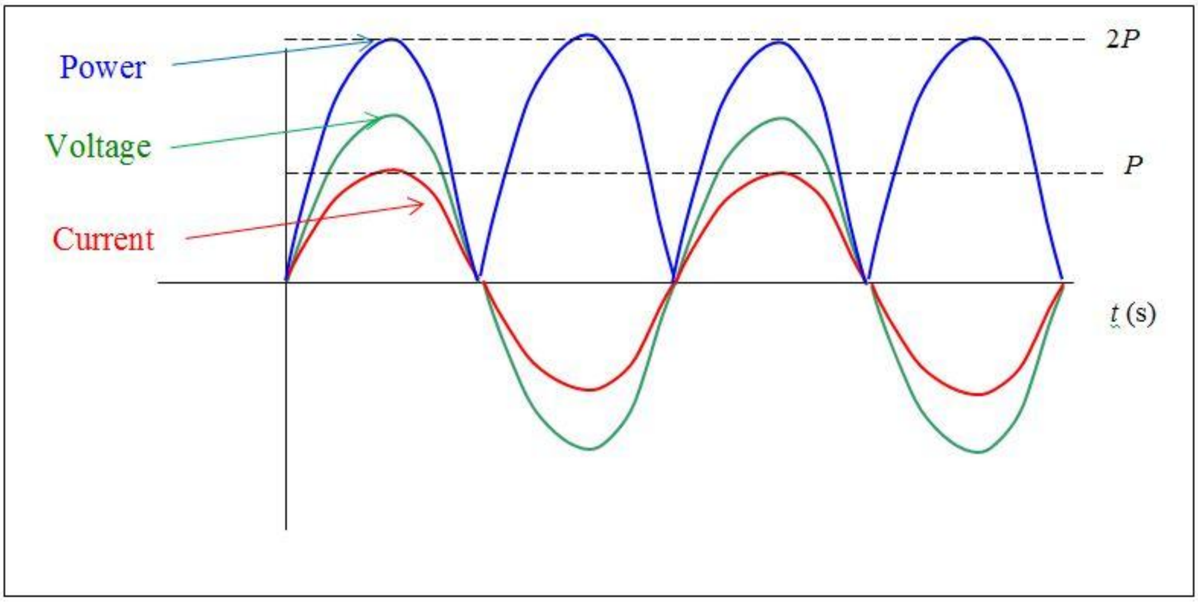


Figure 68 Voltage, current and power in an AC waveform

Notice that power varies from a maximum of $+2P$ to a minimum of 0. Therefore, the average power is P. We never get a negative power, since that would imply that the component was creating energy.

Not every alternating current is a sine wave.

The scale on the *x*-axis is the **period**, not the wavelength.

Tutorial 11.06 Questions

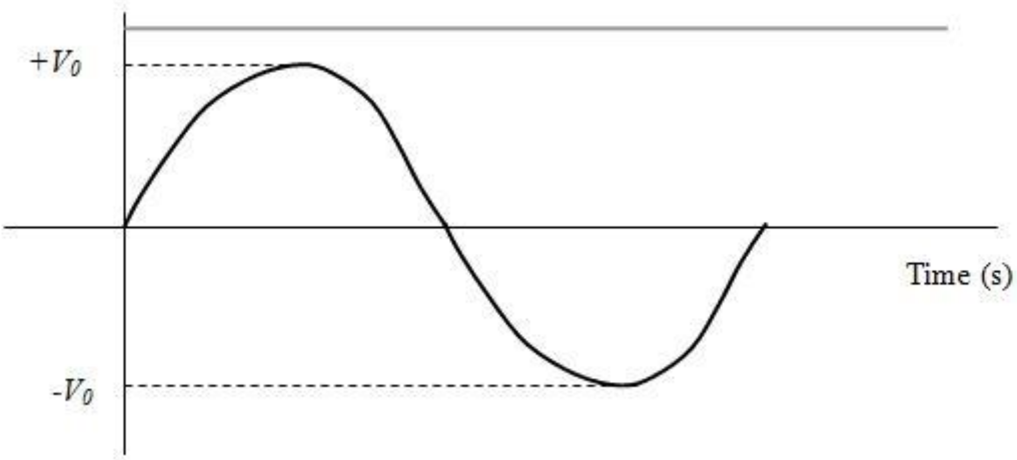
11.06.1

What is the peak voltage of the 230 V ac mains?

11.06.2

On a copy of the diagram below, mark the following:

- AC waveform
- DC
- Period
- Peak Voltage
- RMS voltage



11.06.3

How does the power vary with time if a current of I_{rms} passes through a heater with a voltage of V_{rms}?

Think about the power when V and I are positive and when V and I are negative.

Sketch a graph to illustrate your answer.

How does the peak power compare to the power worked out using the rms values?

Tutorial 11.07 Simple AC Generators	
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11.071 Magnetic Transducer

A generator converts **kinetic** energy to **electrical** energy. It is called an **energy transducer**. A simple DC electric motor will generate DC when it is turned. A pick-up cartridge in a vinyl record deck is a generator (*Figure 69*).

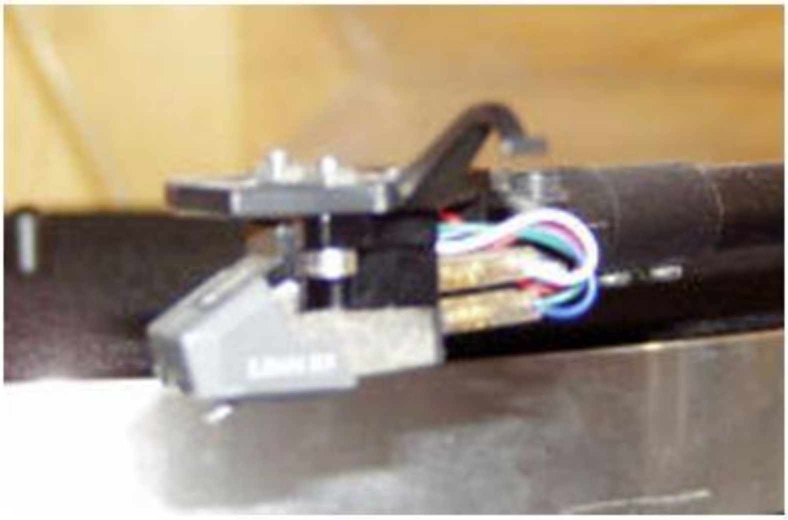


Figure 69 A pick-up cartridge on a record deck

This is a generator that gives off a tiny voltage (100 mV). It is the pickup cartridge on a record deck that plays old-fashioned vinyl LPs (not so old fashioned - HMV report that vinyl record sales are up by 31 % as I write this). The stylus waggles a tiny magnet between four coils. The current is amplified by the amplifier.

The picture below (*Figure 70*) is of an **alternator** that is found on a car engine. Its shaft is driven by a pulley from the crankshaft of the engine

Figure 70 Alternator on a car (Image by Christopher Ziemnowicz, Wikimedia Commons)

The alternator in a car generates 12 volts at up to 70 amps. The AC is converted to DC by diodes to charge up the battery and run the electrical components in a car. Without it, the battery will run down very quickly. Once the battery fades, petrol cars will stop. A similar device is found on light aeroplanes. However, ignition is provided by a separate system of magnetos. Should the battery fail, the plane will not fall out of the sky.

Converting AC to DC is easy. *Figure 71* shows a **bridge rectifier** circuit to make a DC output from an AC input. You can also make a **half wave rectified** DC using a single diode. We will look at this later.

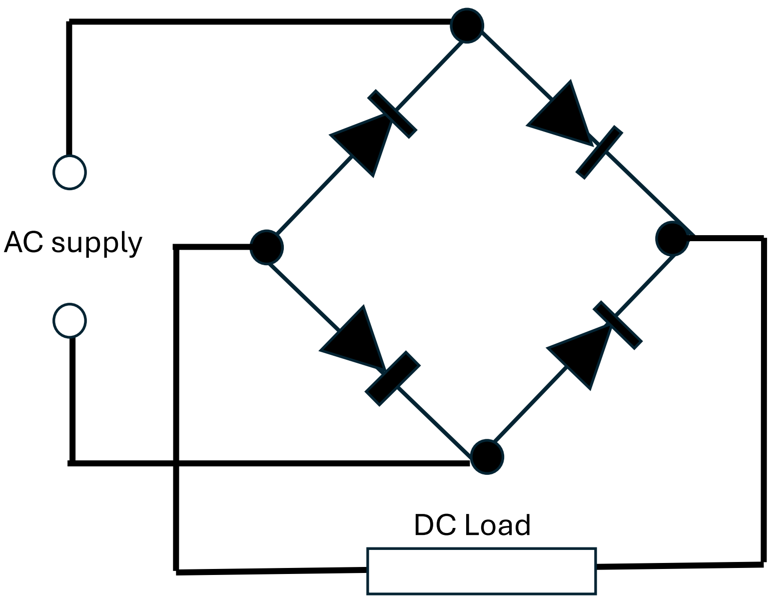


Figure 71 Rectifying AC to DC

11.072 Simple AC Generator

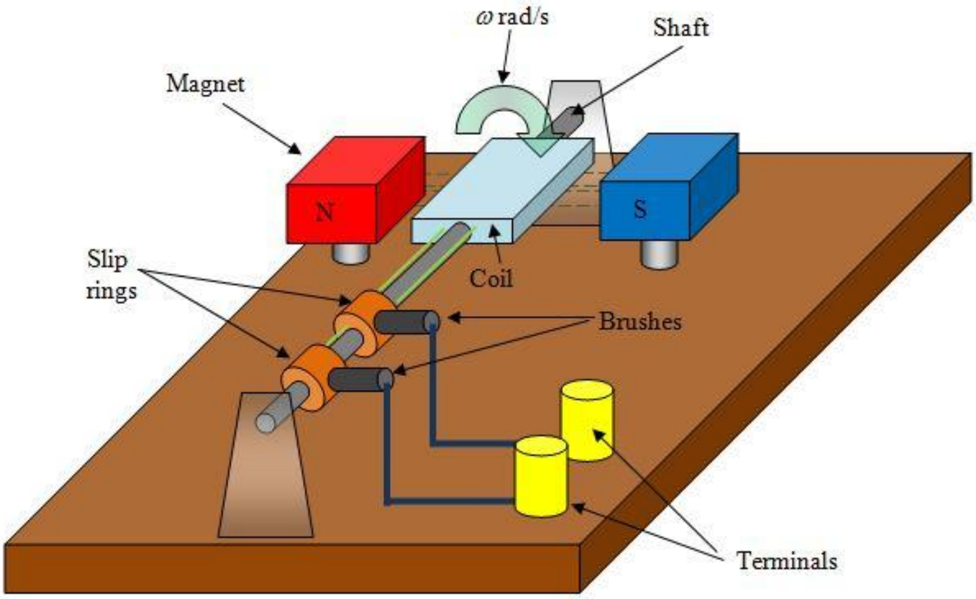


Figure 72 A simple AC generator

The diagram above (*Figure 72*) is a simple **alternating current** generator. It consists of a coil of N turns, radius r and length l spinning in a magnetic field of flux density B. Its angular velocity is ω radians per second. The motion is, of course, circular.

We can make a simple AC generator using the components from the Westminster Motor Kits that are found in many schools and colleges (*Figure 73*).

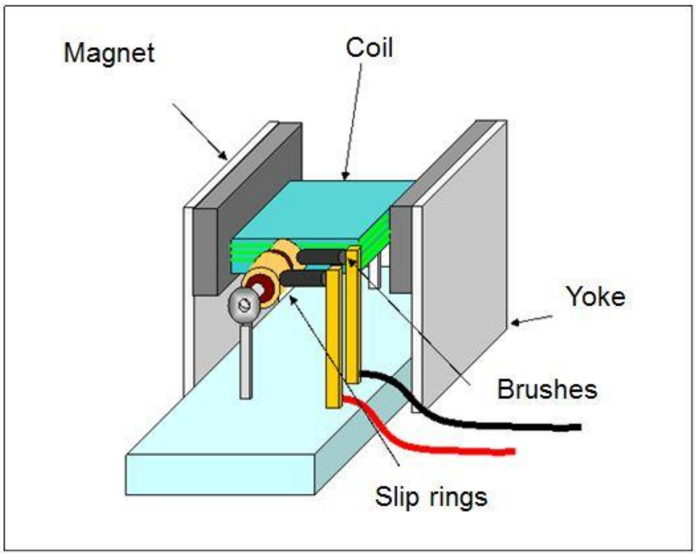


Figure 73 Simple AC generator made from a Westminster motor kit

We can show how the output varies as the generator is turned. The set up is like this (*Figure 74*).

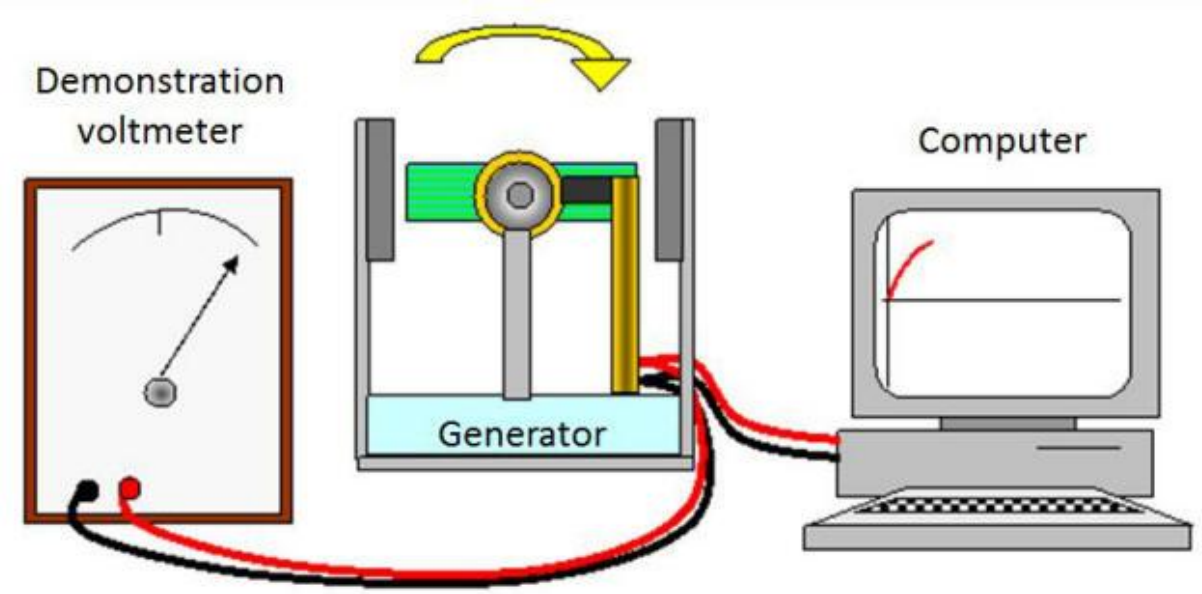


Figure 74 Tracing the waveform from a simple AC generator

We will start the generator from the coil being vertical. The red spot allows us to trace the motion of the coil (*Figure 75*).

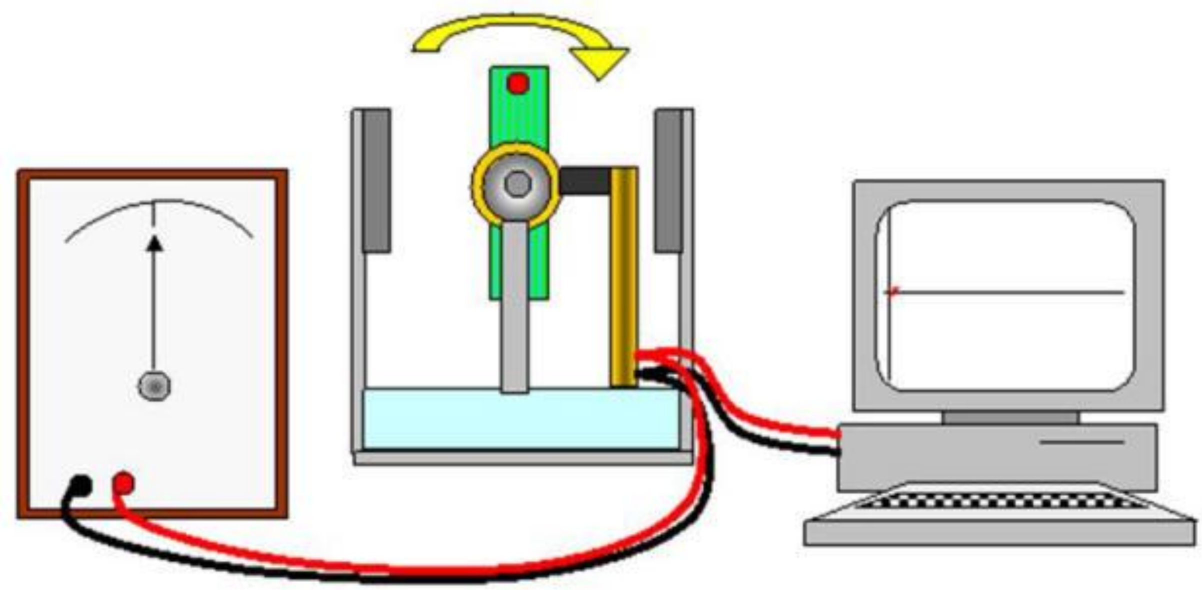


Figure 75 Starting with the coil vertical

A quarter of a turn later (*Figure 76*):

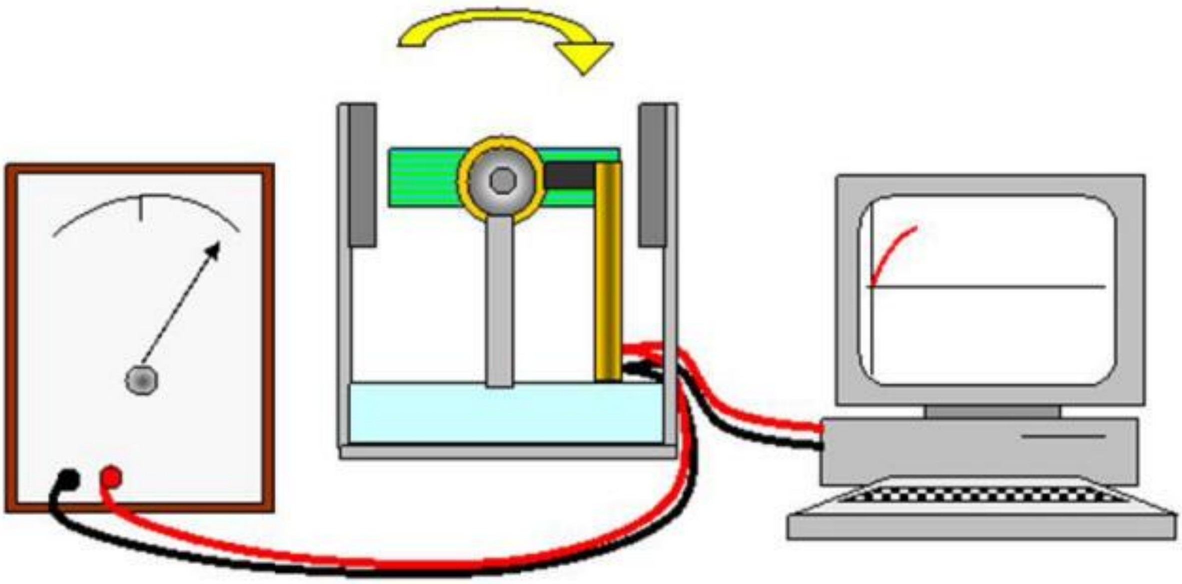


Figure 76 After a quarter turn

Half a turn later (*Figure 77*):

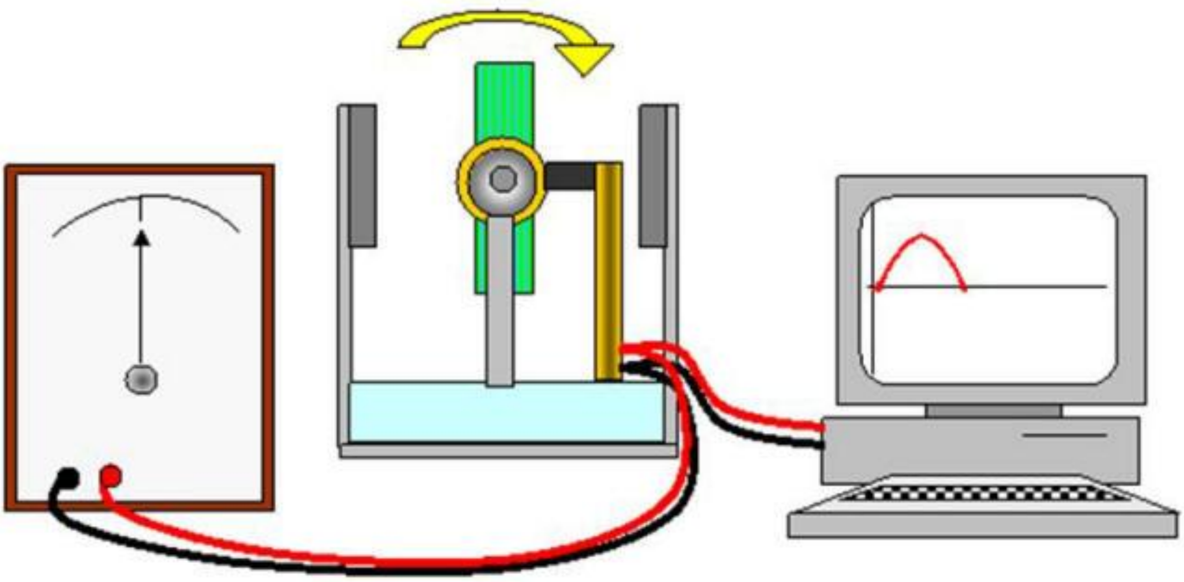


Figure 77 After a half turn

Now at three quarters of a turn (*Figure 78*):

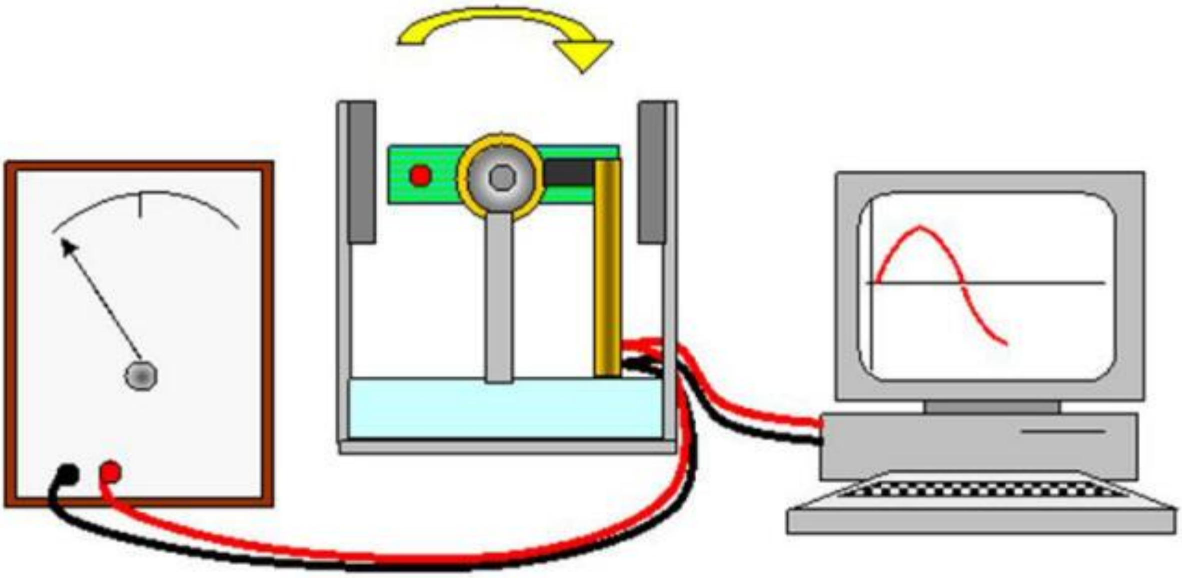


Figure 78 After three quarter of a turn

And now a full turn (*Figure 79*):

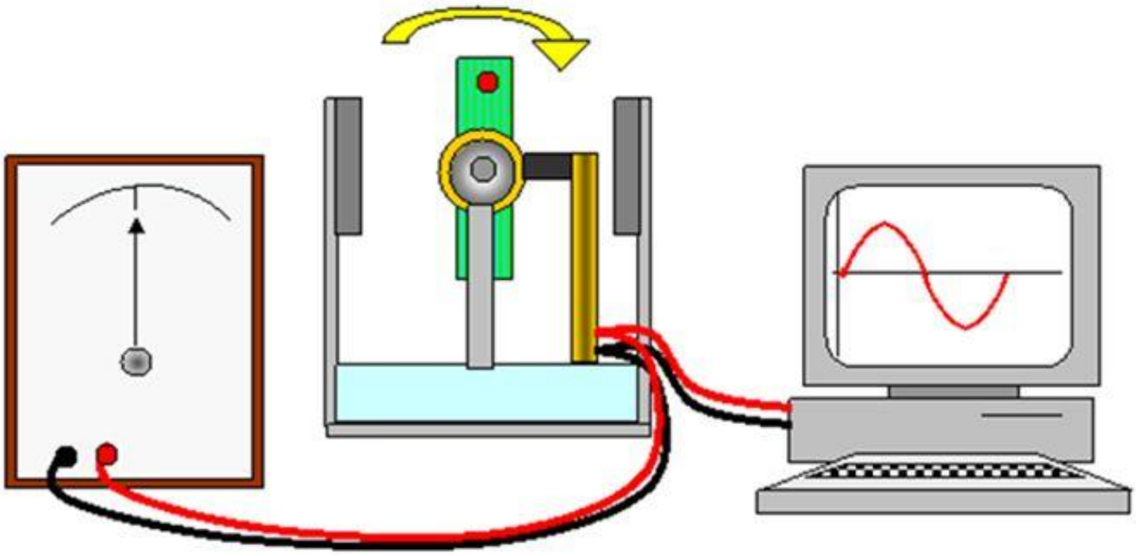


Figure 79 After a whole turn

The trace shows a **sine wave**.

11.073 Explaining the sine wave

We can use the fact that circular motion and simple harmonic motion are closely linked. So, we can use the equation for displacement:

$$x = A\cos \omega t \quad \text{………….. Equation 72}$$

Here the x term refers to the **displacement** from a fixed point. We can make the point at which the coil is vertical the "rest position". The maximum amplitude is r as in the diagram (*Figure 80*).

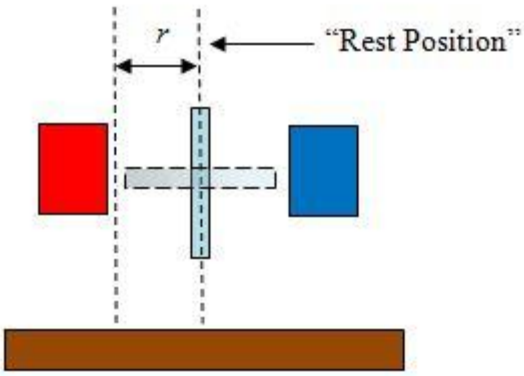


Figure 80 Coil in the "rest position"

So, our equation becomes

$$x = r\cos \omega t \quad \text{……………… Equation 73}$$

Since the coil is rotating at a constant angular velocity, ω, the speed of the edge of the coil is given as:

$$v = \omega r \quad \text{…………… Equation 74}$$

From SHM we can say:

$$v = -r\omega \sin \omega t \quad \text{............ Equation 75}$$

We can now bring in our EMF and linear speed equation:

$$E = -NvwB$$ .... *Equation 76*

We can combine the *Equations 75* and *76* above to give:

$$E = -N(-r\omega \sin \omega t)lB$$ .... *Equation 77*

The minus signs disappear, and we can also say that:

$$A = rl$$ .... *Equation 78*

Therefore:

$$E = BAN\omega \sin \omega t$$ .... *Equation 79*

This explains why the output of an AC generator is sinusoidal. The output of an AC generator is a **sine wave** (*Figure 81*).

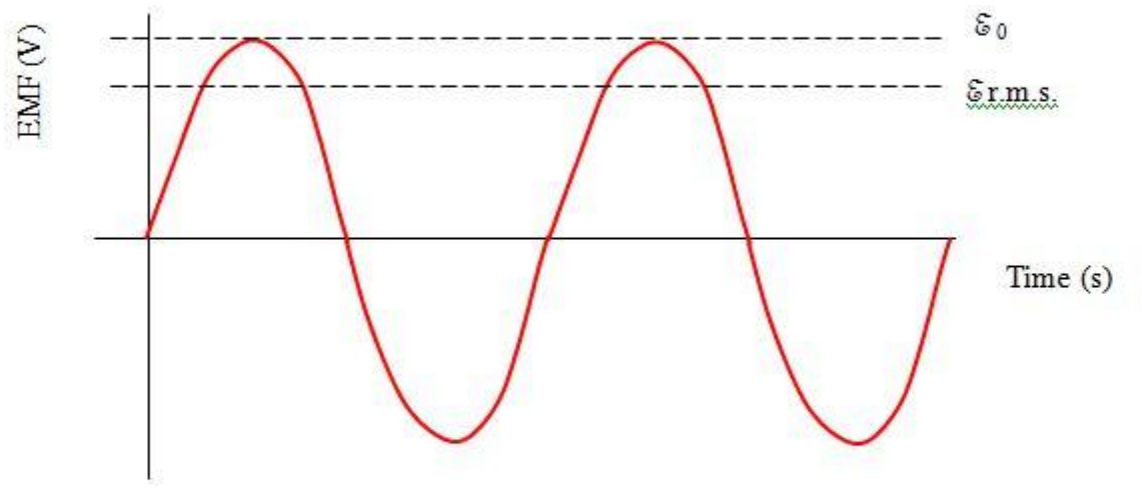


Figure 81 Output of an AC generator is a sine wave

The maximum value of the e.m.f. is when $\sin \omega t = 1$. Therefore:

$$\mathcal{E}_0 = BAN\omega$$ .... *Equation 80*

11.074 Alternators

The simple AC generator described above is inefficient but could be made more efficient by changing the shape of the magnet and wrapping the coil around soft iron. Practical AC generators have a rotating magnet (**rotor**) which passes between stationary coils (**stator**). The alternating e.m.f is induced in these coils. The machine is called an **alternator**.

Here is a generator that generates single phase alternating current (*Figure 82*).

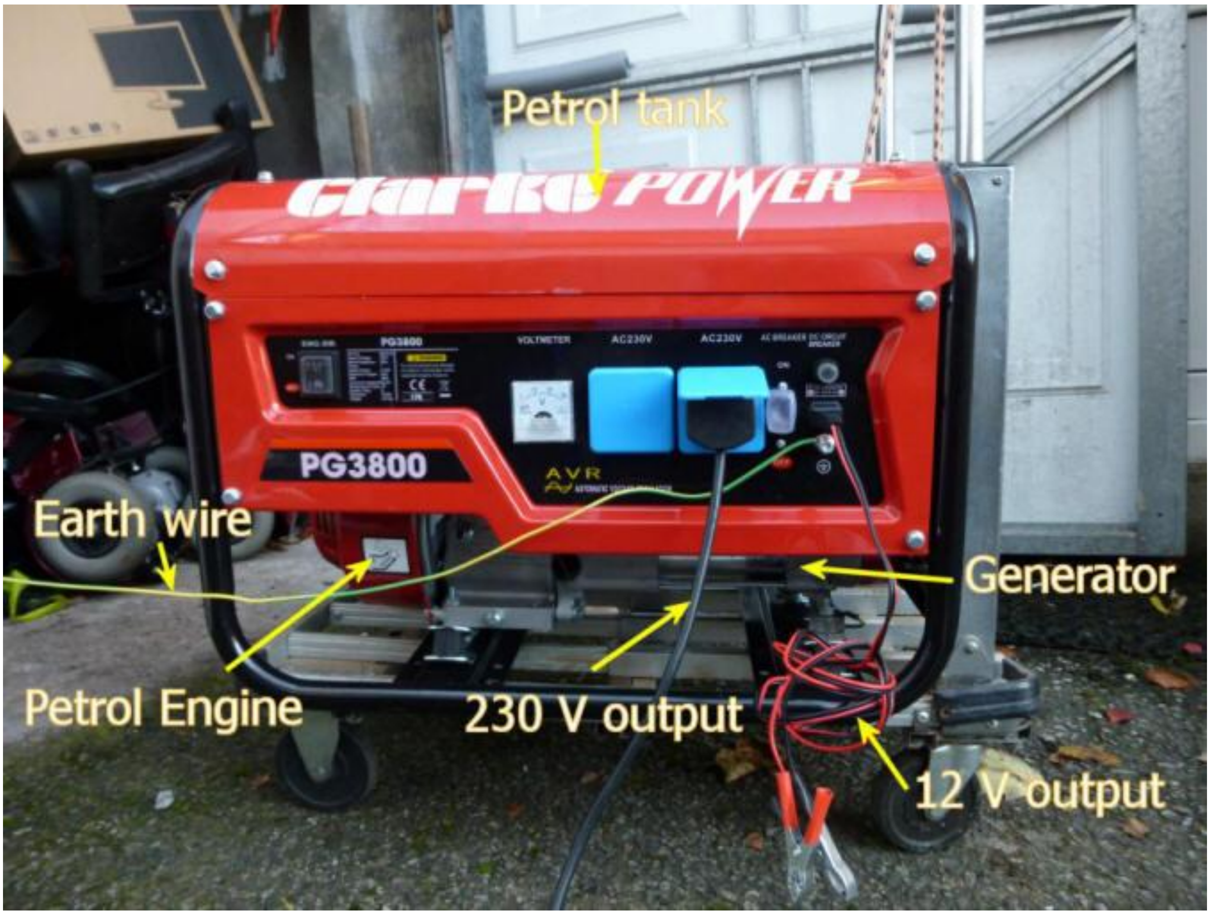


Figure 82 A petrol generator

A 7 PS (pferdstärke or horsepower) petrol engine drives this generator which provides power to a house when there is a power cut. It also can be used to drive electric garden machinery. The generator can be seen just below the front panel. As well as a 230 V output, there is a 12 V output from the **exciter**. This is a separate low voltage DC generator that generates electrical energy for the electromagnet in the **rotor**. The rotor turns between stationary coils. These produce the 230 V output. The stationary coils are referred to as the **stator**.

Power station generators are massive. They have a rotor that is connected to its own generator, called an **exciter**. The stator coils are placed at 120 degrees to each other to allow **3-phase AC** to be generated. The voltage is 25 000 V, while currents of 15 000 A are common. The whole machine is cooled by hydrogen gas, which has a particularly high specific heat capacity. The picture (*Figure 83*) below shows a power station alternator.

Figure 83 A power station alternator

The generator is actually in the rectangular box on the right. To the left is the low-pressure turbine. Turbines and generators are so big that when the machine is off, the shaft has to be rotated slowly, otherwise it would sag and go out of shape (which is not a good idea). It is driven by a **barring motor** (*Figure 84*).

Figure 84 A barring motor

Power station alternators generate electricity using **3-phase**. This means that there are **three separate live wires** that carry the electrical energy away from the machine. There is a common neutral wire. The study of three-phase AC is not part of any A-level syllabus.

Tutorial 11.07 Questions

11.07.1

A coil of 10 turns and area 6 cm^2 is turning at a rate of 500 r.p.m. between the poles of a magnet of magnetic field strength 0.45 T. What is the peak voltage? What is the r.m.s voltage?

11.07.2

The maximum power that can be given out by this generator is 3 kW. If the engine output is 7 PS, calculate the efficiency of the generator.

(1 PS = 750 W)

Tutorial 11.08 The Cathode Ray Oscilloscope	
All Syllabi	
Contents	
11.081 The CRO	11.082 Different Kinds of Waves on the CRO
11.083 How the CRO Works (Extension only)	

11.081 The CRO

The **cathode ray oscilloscope** (CRO) is the electronics engineer's best friend. It is an instrument that displays alternating waveforms. These can be simple, such as the sinewave in alternating current. Or the waveforms can be complex, such as the pattern made by someone's voice. Cathode ray oscilloscope is a bit of a mouthful, so it is shortened to **CRO**.

You are NOT expected to describe how it operates, but you are expected to be able to set it up and interpret displays on the screen.

The CRO is connected in exactly the same way as a **voltmeter**, i.e. in **parallel** with a component. The input resistance is very high indeed and the electron beam acts as a pointer of negligible inertia. It is also robust and not easily damaged by overloading.

The CRO can be used as a DC voltmeter (*Figure 85*). We get a horizontal line or a dot, depending whether the **time base** is on. If it is used as an AC voltmeter, it will show the sinusoidal waveform. It will, of course show more complex waveforms of audio signals, but we won't worry about these.

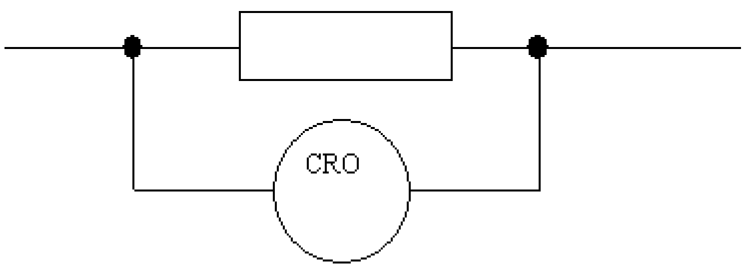


Figure 85 CRO used as a voltmeter

The CRO can also be used to measure the voltage across a resistance of known value. Therefore, it can be used as an **ammeter** (*Figure 86*).

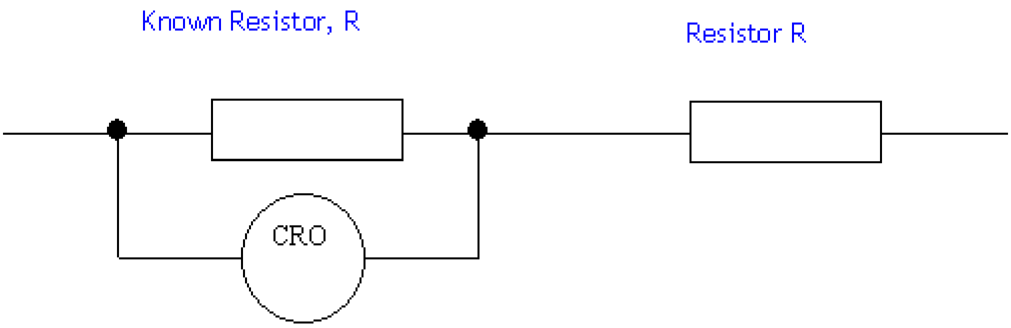


Figure 86 CRO used as an ammeter

The CRO is shown in the picture below (*Figure 87*)

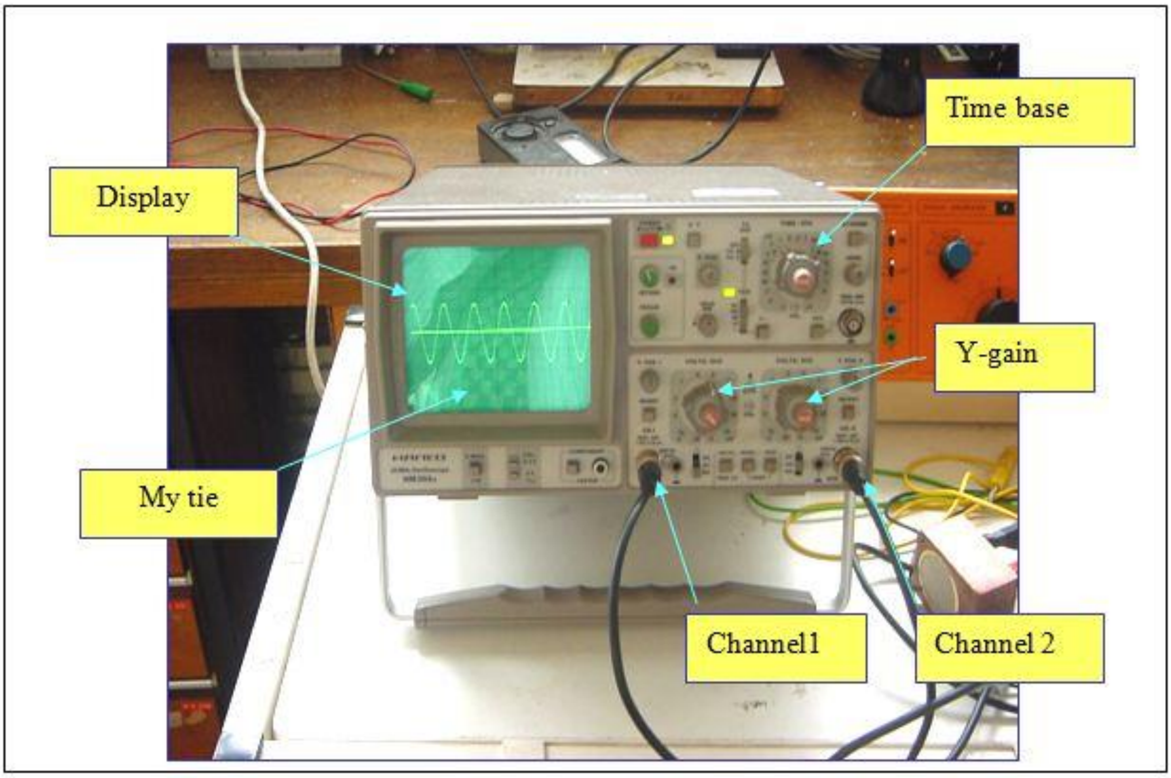


Figure 87 A typical CRO in a college lab

The most important controls that we use are:

- The **vertical sensitivity**, voltage gain, or y-gain setting, calibrated in volts per centimetre (V cm^{-1}).
- The **time base**, in seconds per centimetre (s cm^{-1}).

The CRO is a perfect voltmeter as its input resistance is very high indeed.

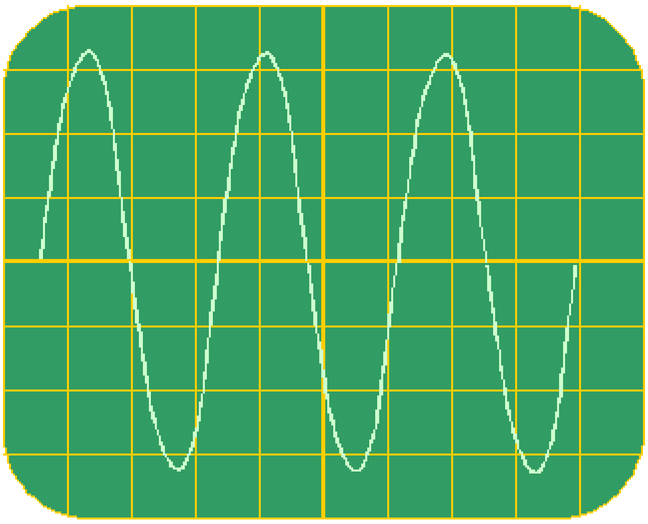
The horizontal display represents the period. It is NOT the wavelength.

Remember:

- We measure the **voltage** on the **vertical axis**. We can adjust the sensitivity by turning the knob marked **y-gain** or **voltage gain**.
- The horizontal direction is determined by the **time base** setting. We can change this by using the **time base** knob.

As well as analysing the waveform, there are two measurements we can make with the CRO (*Figure 86*).

- We can determine the peak voltage of the AC waveform shown below.
- We can also read the period, which in turn allows us to work out its frequency.



- The y-gain is set at 2 V/cm
- The total height of the wave from peak to trough is 6.4 cm

$\Rightarrow V_{pk\ to\ pk} = 12.8$ V
$\Rightarrow V_0 =$ **6.4 V**

- The time base is set at 0.5 ms/cm.
- 1 cycle occupies 2.9 cm

$\Rightarrow T = 1.45$ ms $=$ $\mathbf{1.45 \times 10^{3}}$ s

$\Rightarrow$ Frequency $= 1 \div 1.45 \times 10^{-2}$ s
$=$ **690 Hz**

Figure 88 Reading the CRO screen

Notice that:

- The **peak to peak voltage** is 12.8 V. Often engineers read the **peak to peak voltage** off the CRO as the determination of the 0 level is not always easy. The **peak to peak voltage** is **half** of the peak to peak voltage.
- The root mean square voltage, which we use in electrical calculations, is the peak voltage divided by √(2)
- Therefore, the V_{rms} = 6.4 ÷ √2 = **4.5 V**

11.082 Different Kinds of Waves on the CRO

Not every waveform displayed on a CRO is sinusoidal. The picture below shows a **square** waveform (*Figure 89*):

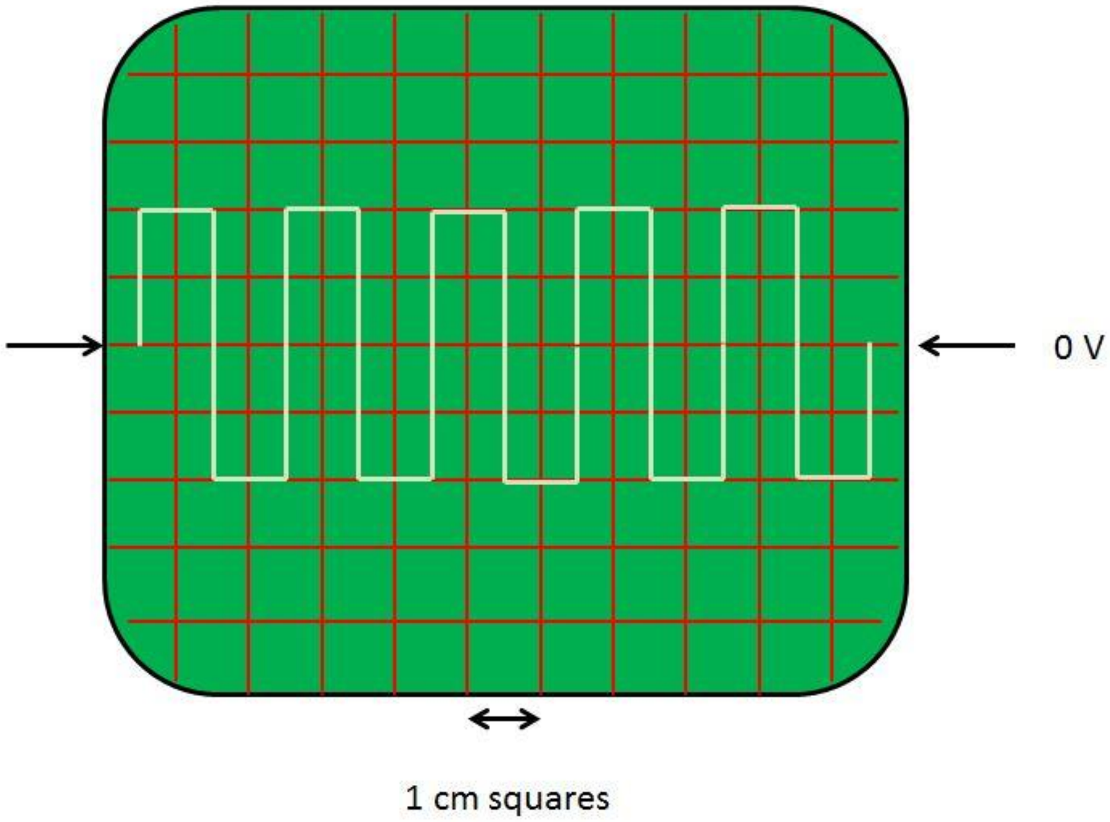


Figure 89 Square AC waveform on a CRO screen

With a square wave, there is no RMS voltage.

Some waveforms are not alternating, but **unidirectional**. Now look at this picture (*Figure 90*).

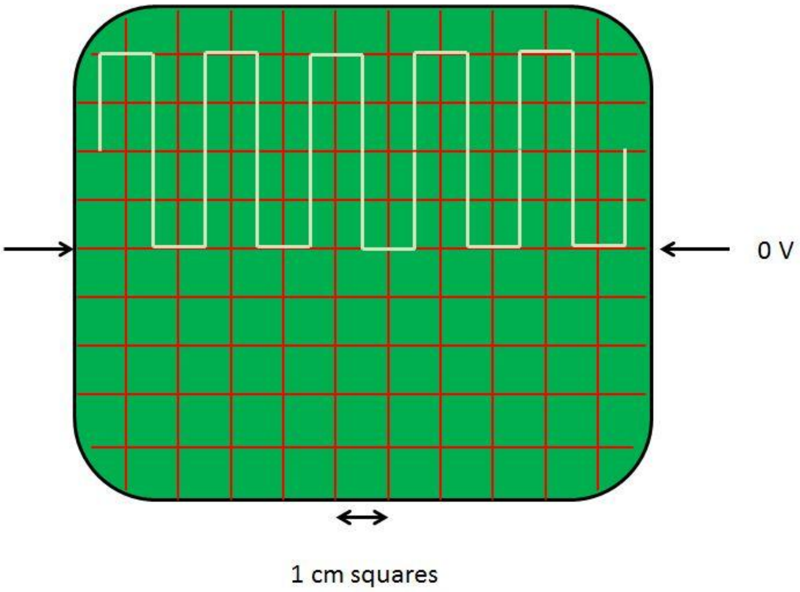


Figure 90 Unidirectional waveform

There are more waveforms that are not sinusoidal. This is a **triangular** alternating waveform (*Figure 91*).

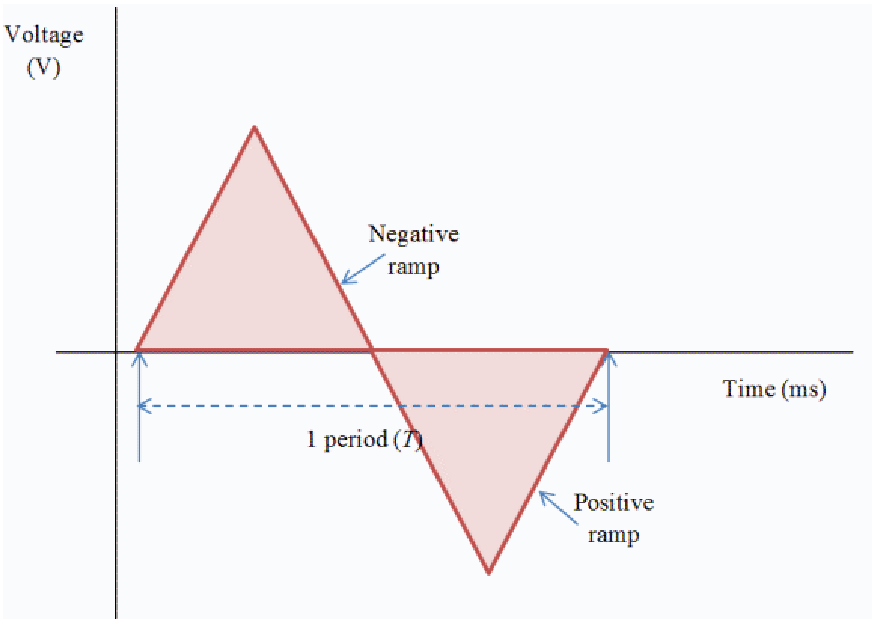


Figure 91 A triangular AC waveform

This waveform is a saw-tooth waveform (*Figure 92*). It is unidirectional.

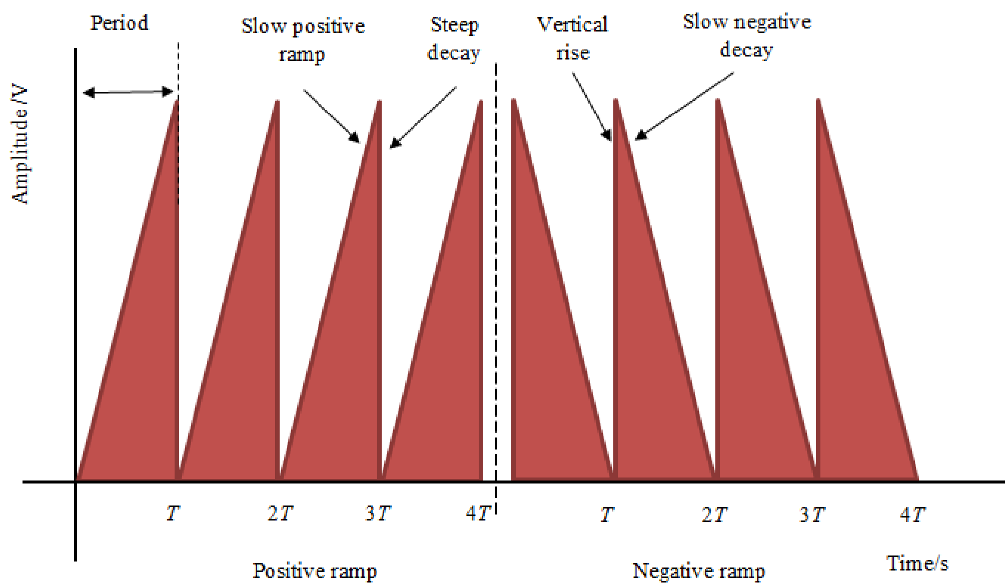


Figure 92 A unidirectional sawtooth waveform

Note that the first set of waves has a **positive ramp**. The second set has a **negative ramp**. You have to have one or the other.

The picture below shows a **complex** alternating waveform (*Figure 93*).

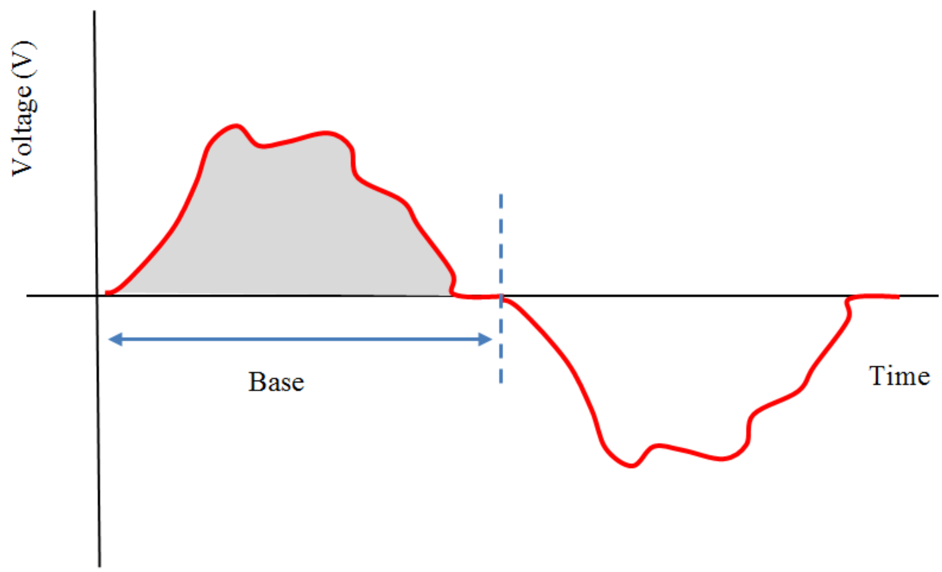


Figure 93 A more complex waveform

To study a complex waveform, you really need to have a **storage oscilloscope**, which is one that has a memory. A normal CRO will display a waveform for as long as it is generated. As soon as the source of waves is turned off, the waveform will no longer be displayed.

Nowadays it's possible to buy a CRO sensor that connects to a computer in the same way as a data logger (*Figure 94*).

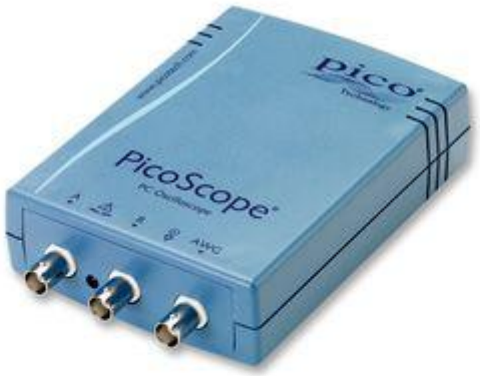


Figure 94 A CRO sensor for a data logger

11.083 How the CRO works (Extension only)

In the A-level (or equivalent) examination, you are NOT expected to discuss the way a CRO works. However, at university level, an understanding of the way a CRO works will be helpful.

Before you read this, you need to understand how electrons behave in magnetic fields and electric fields.

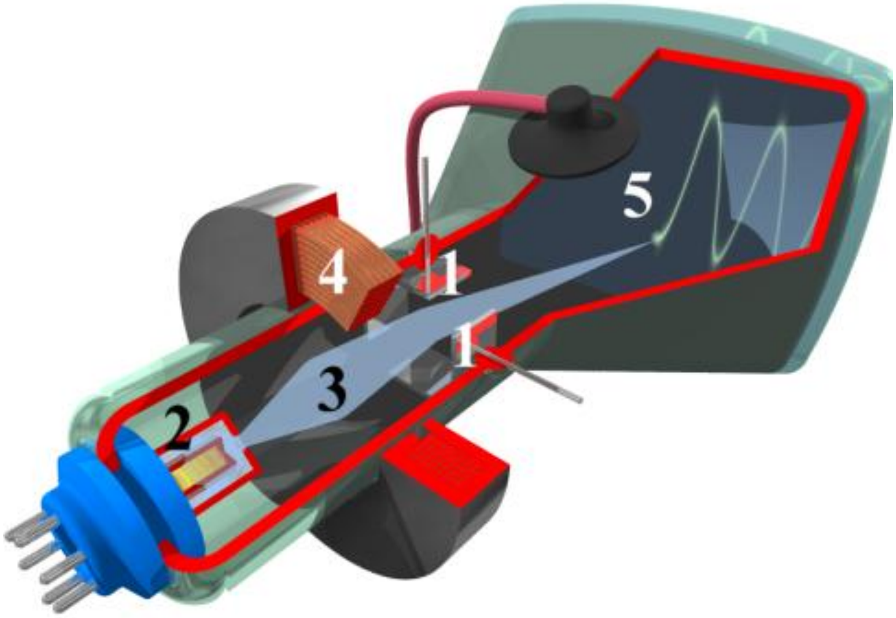


Figure 95 Diagram of a CRO tube (Diagram from Wikimedia Commons)

Refer to *Figure 95*. Electrons are boiled off the filament of the **electron gun** (2) and are **accelerated** by the anode to form a **beam** (3). The beam is focussed by a **magnetic coil** (4) to give a sharp image on the **phosphor scree**n (5). When you adjust the **focusing knob** on the control panel, you are adjusting the current through this coil. Another pair of coils allows you to adjust the **position** of the beam on the screen. (This can be quite temper-trying!)

The beam passes through the **deflection plates** (1) which produce a uniform electric field. The plates are arranged like this (*Figure 96*).

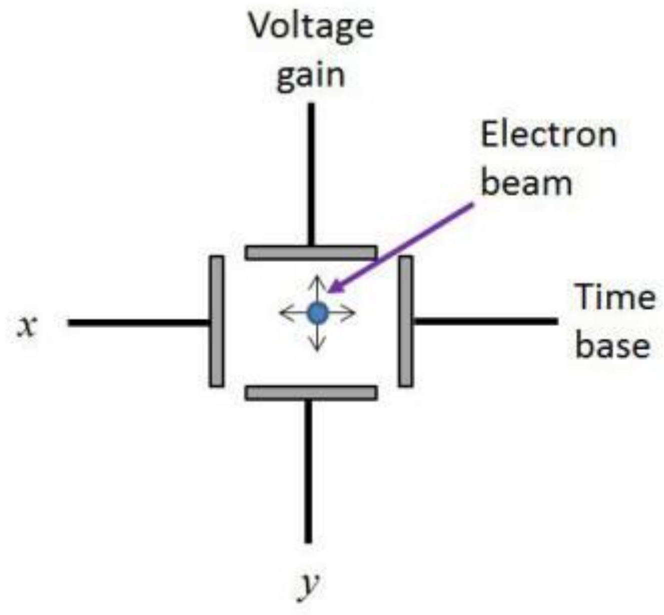


Figure 96 Deflecting the electron beam

The **x-plates** respond to the **time base** input. If there is no input voltage, we see *Figure 97*.

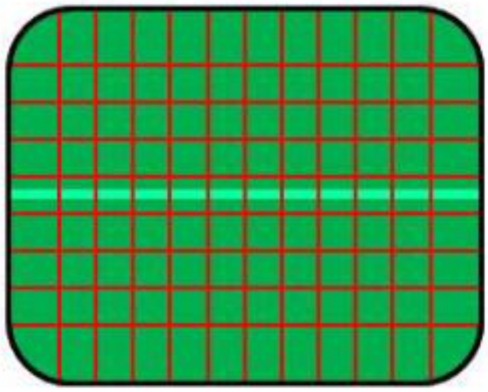


Figure 97 Time base with no input voltage

If there are ten 1 cm squares across the screen, and the time base is set to 10 ms cm^{-1}, the electron beam will take 100 ms to cross the screen. Therefore, the frequency is 1 ÷ 0.100 s = 10 Hz. This is a slow sweep, and you will see the beam (a spot) crossing the screen.

If we turn the time base off and apply an **alternating voltage**, we see a **vertical** line, as the y-plates respond to the **voltage gain** input. They are deflecting the electron beam vertically, so we see *Figure 98*.

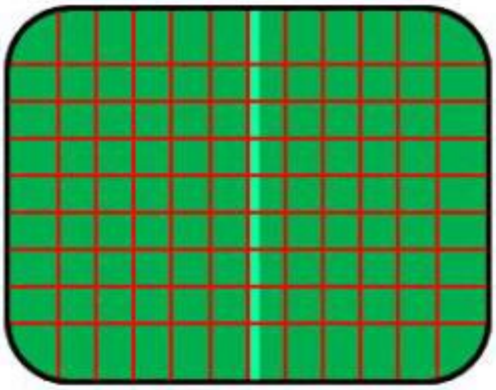


Figure 98 Voltage applied but time base switched off

The beam is **vertical**.

If neither set of plates is switched on, we see a **spot** in the middle of the screen (*Figure 99*).

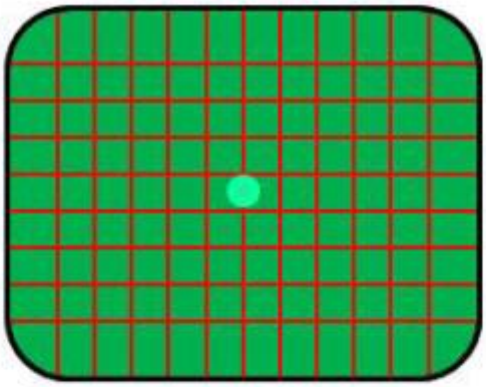


Figure 99 No voltage and no time base

If we apply a **positive** DC voltage, the spot moves **upwards**:

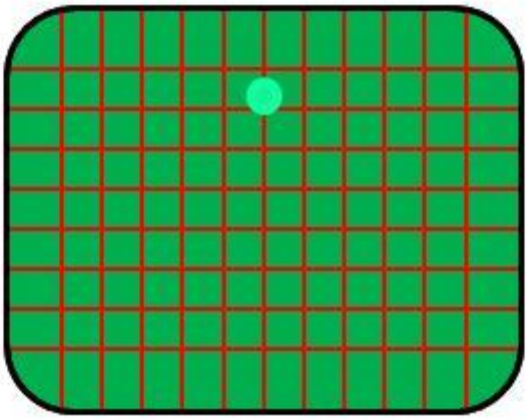


Figure 100 Time base off and positive DC voltage

The spot moves downwards if the voltage is negative.

If the voltage is a positive DC voltage <u>and</u> the time base is turned on, we see a straight line above the centre like this (*Figure 101*).

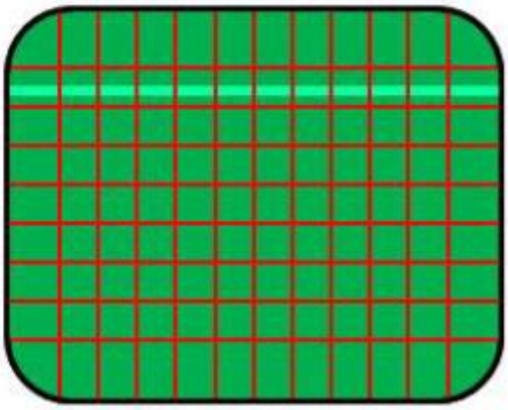


Figure 101 DC voltage applied, and time base switched on

If the DC voltage is negative, the line will be below the centre line.

If the time base is turned on, <u>and</u> an **alternating** voltage is applied, we get a **sine wave**:

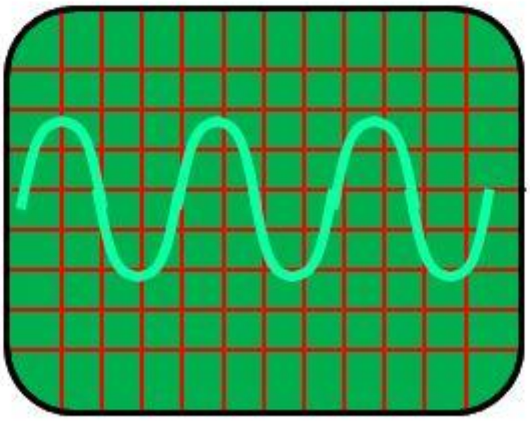


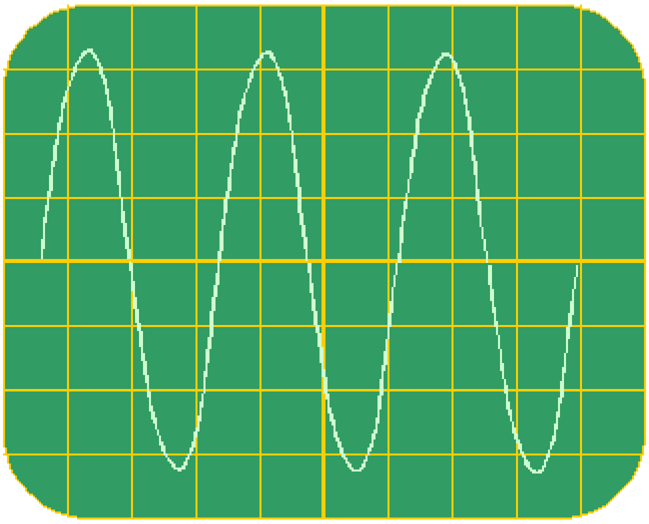
Figure 102 Time base on and alternating voltage

The y-plates are connected to the output of amplifier that amplifies the voltage from about10^{-6} V to several hundred volts to make the field between the plates. The x-plate drivers sweep the beam across the screen at a frequency as high as 10^{6} Hz.

Tutorial 11.08 Questions

11.08.1

Look at the CRO display:



The time base is set at 2 ms/cm and the y gain at 0.5 V/cm

(a) What is the peak to peak voltage?

(b) What is the peak voltage?

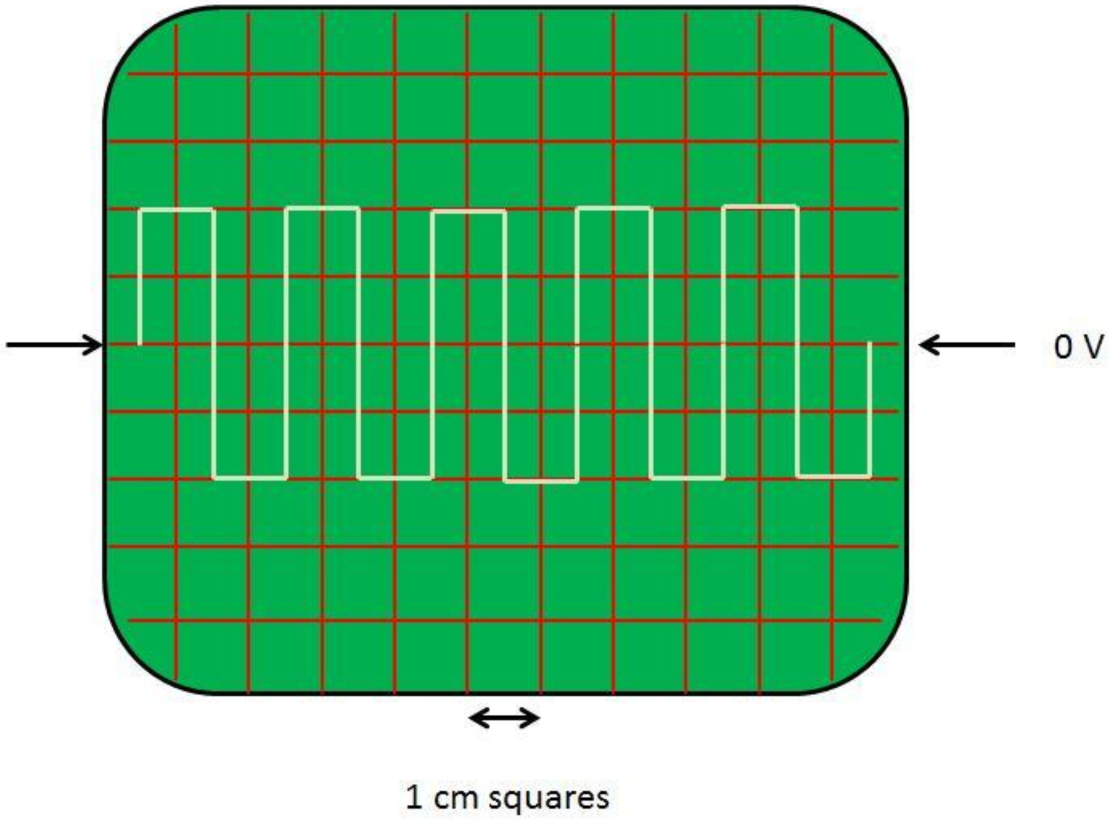
(c) What is the rms voltage?

(d) What is the period?

(e) What is the frequency?

1.08.2

Look at the CRO display:



The time base is set at 2 ms cm^{-1} and the voltage gain is 2.0 V cm^{-1}.

(a) What is the period?

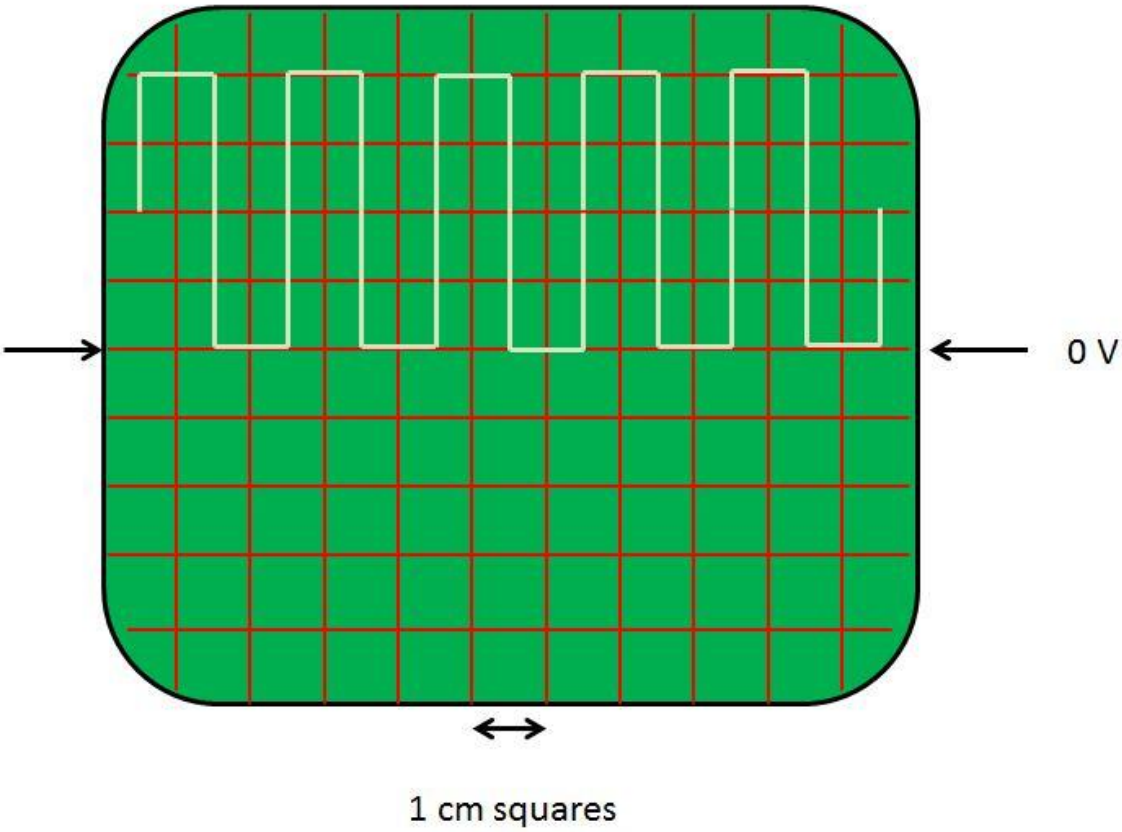
(b) What is the maximum voltage?

(c) What is the minimum voltage?

(d) Is this an alternating waveform?

11.08.3

Look at the CRO display:



The settings on the CRO are the same as they are in Question 11.08.2.

(a) What is the maximum voltage?

(b) What is the minimum voltage?

(c) Is this an alternating waveform?

3. The Transformer Effect

Tutorial 11.09 Transformers

All Syllabi

Contents

11.091 Inducing a Voltage

We can use a magnetic field to induce a voltage in two ways:

1. Relative movement. The size of the voltage depends on:

- Speed the magnet passes through a coil or vice versa.
- Number of turns in the coil.
- Strength of the magnet.

2. Changing a magnetic field. We don't have to make the magnetic field move. If we turn the current on or off, there is a change in the magnetic field, and that induces a voltage in a second **unconnected** coil. This is called the **transformer effect** or **mutual induction**.

Consider this set up:

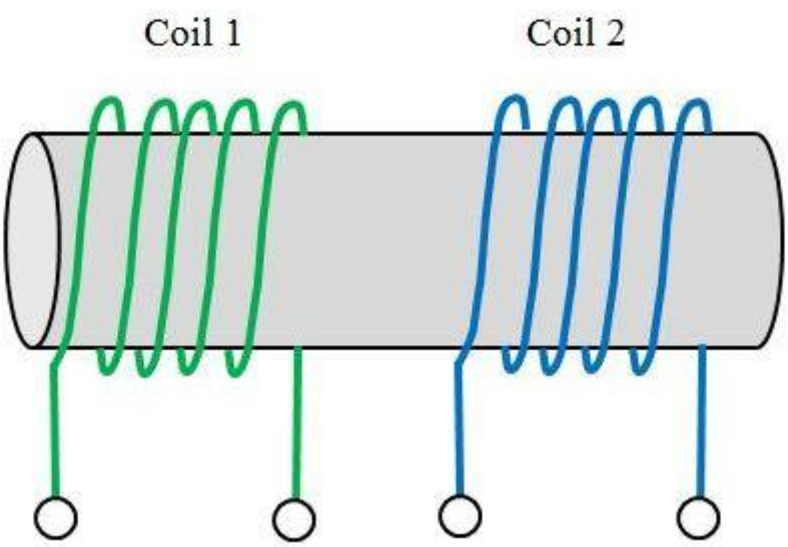


Figure 103 Mutual induction

If a changing current I_1 A is passed through Coil 1, there will be a changing voltage $\mathcal{E}_2$ *V* in Coil 2. This can be worked out with the formula:

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}$$

.... *Equation 81*

11.092 Transformers

Electric locomotives take their power from a variety of systems. In most modern electrified railways, the power is supplied to the locomotive by a 25 000 V overhead line at 50 Hz ac. However, the motors work at about 1500 V, so there needs to be a way of reducing the voltage. This could be done using a resistor, but it would be extremely wasteful. So, it's done with a **transformer**.

Figure 104 This locomotive takes its current from a 25 kV overhead wire

The transformer reduces the voltage of 25 kV in the overhead power line to about 1500 V for the traction motors that turn the wheels, The **transformer** is a machine that is simplicity itself. It consists of:

- A **primary coil** connected to the alternating power source. This provides the changing magnetic field.
- A **secondary coil** connected to the load.
- A **laminated soft iron core**.

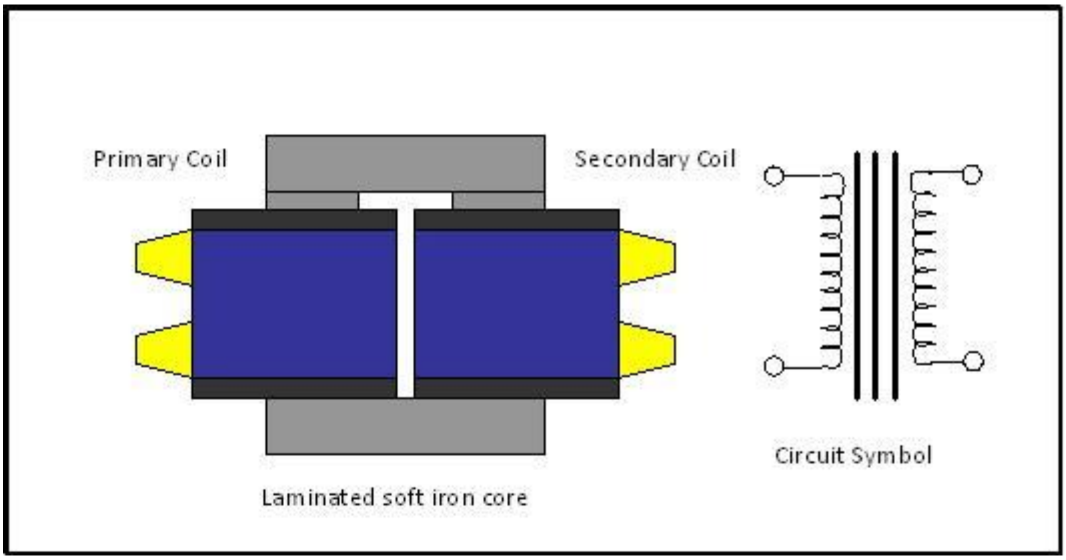


Figure 105 Construction of a transformer and its circuit symbol

The two coils are electrically *completely* different circuits. No current can pass between either coil. Either of the coils can act as a primary.

The **laminated** core is made up of layers of soft iron separated by an insulating layer of varnish or glue. This reduces losses from eddy currents. **Soft iron** is NOT soft like putty; it is heavy and hard. However, "soft" means that it loses its magnetism immediately the current is turned off. Therefore, the magnetic field can change forwards to backwards as the current changes.

The ratio of the input voltage to the output voltage is the same as the ratio of the number of turns on the primary to the number of turns on the secondary. We can write this as:

$$\frac{\text{Number of turns on the primary}}{\text{Number of turns on the secondary}} = \frac{\text{Primary Voltage}}{\text{Secondary voltage}}$$

In Physics code:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$ ………… *Equation 82*

If N_1 is greater than N_2, we have a **step-down** transformer, because the voltage is reduced. A **step-up** transformer increases the voltage.

If a transformer is 100 % efficient (and it nearly is) we can say that:

power in = power out

$$V_1 I_1 = V_2 I_2$$ …………… *Equation 83*

Therefore, we can say that when the voltage is lower, the current is bigger. We can rewrite the transformer equation in terms of current to give us:

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$ .... *Equation 84*

In practice, the transformer is about 97 % efficient. When a large transformer is transferring a lot of energy, even 3 % losses produce a fair amount of heat. Therefore, the transformer is cooled with oil which is pumped to heat exchangers.

Figure 106 Large transformer at a power station

The picture above (*Figure 106*) shows the huge transformers used at a power station. You can see the massive cables coming out from the generator (they are like pipes). In this power station, the generator voltage is 15 000 V, and this transformer steps the voltage up to 132 000 V. Note that there three cables. This is because the current is **3-phase**. You are NOT expected to know about three phase in the exam, other than its being used in electricity transmission.

The advantage of a three-phase system is that the current in each of the phases is 1/3 of what it would be if the current were single phase.

Transformers vary in size from the huge machines as shown the *Figure 106*, to tiny devices that go on printed circuit boards. Many devices with rechargeable batteries have transformer, often in the plug itself. *Figure 107* shows an electric toothbrush that mounts to a base station. The casings are plastic, so there can be NO passing of electricity.

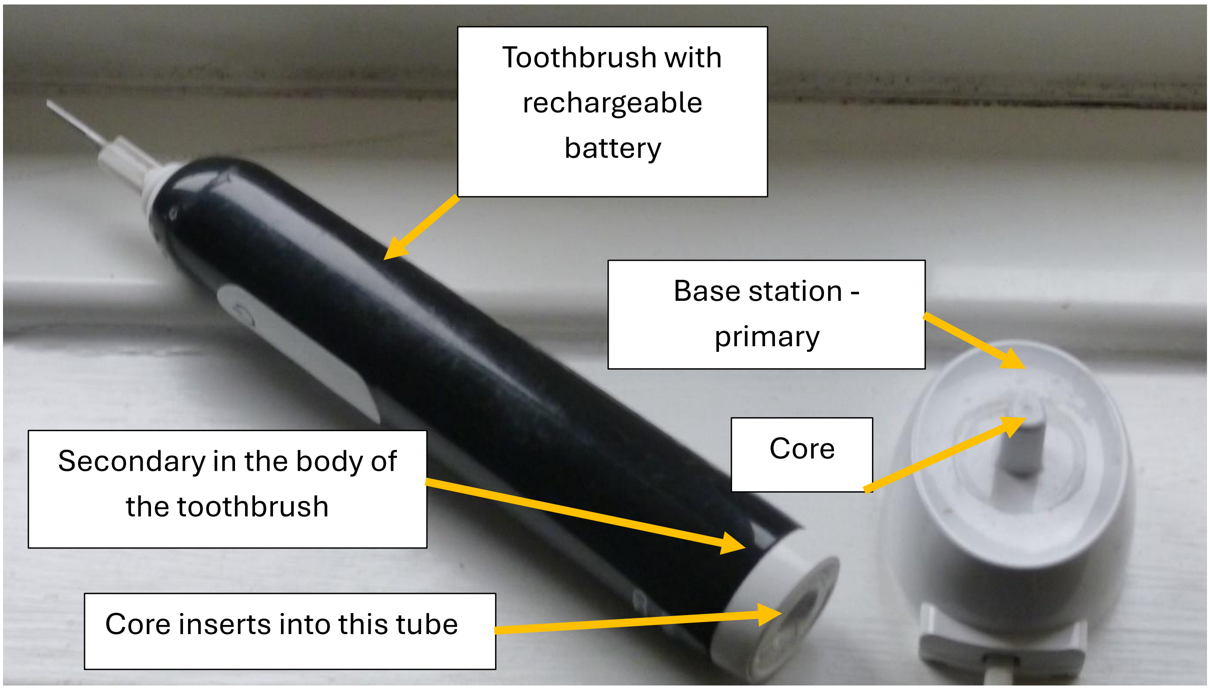


Figure 107 A rechargeable electric toothbrush

The base station has a primary coil which plugs into a mains source, e.g. a razor socket (which is itself a 1:1 transformer). The base station has a core built in that also acts as a mounting pin, which in turn is inserted into a short tube. There is a secondary that surrounds the tube, which in turn provides a supply to the rectifier and regulator circuit for the battery. The transformer system is not particularly efficient, but that does not matter that much to the user, who will use it about twice a day.

10.093 What are the sources of inefficiency in a transformer?

- The coils will have a certain value of **resistance**. If the currents are large, the energy loss is large, since $P = I^2R$.
- With the laminated soft iron core, there are still **eddy currents**, even though they are much reduced. These will heat up the core.
- Work has to be done in building up the magnetic field in the core. It takes energy to line up all the domains. The energy recovered as the magnetic field falls is less than what is put in. This is called **hysteresis**, shown on the graph below (*Figure108*).

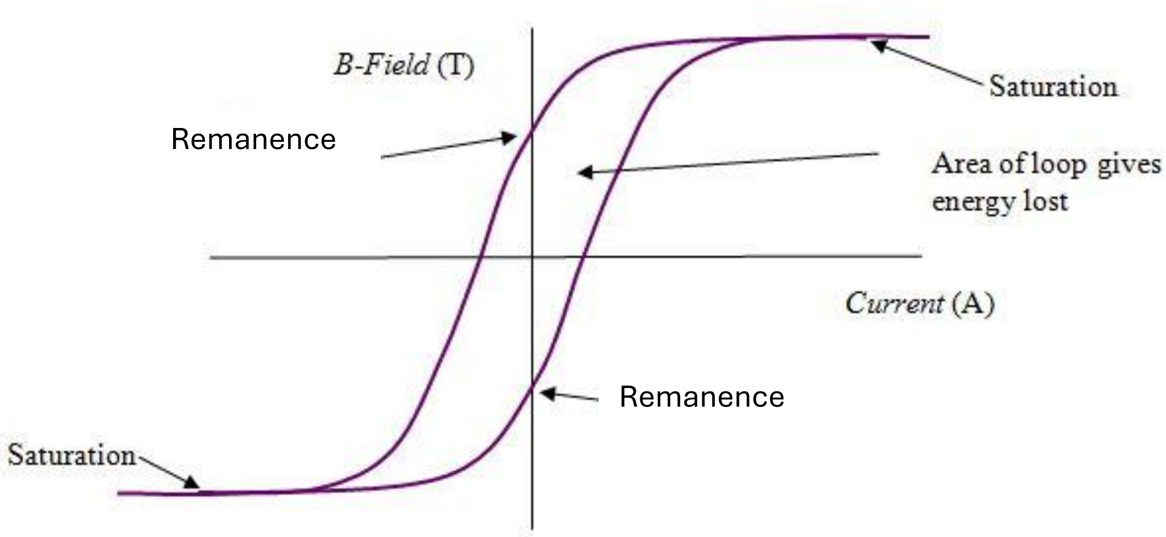


Figure 108 Hysteresis in a transformer core

A few points to note:

- The area within the loop is the energy lost.
- All magnetic materials have a certain **remanence**, meaning that the material is magnetised, even when the current is zero. In soft iron, the remanence is low.
- There is a value of current at which all the tiny molecular magnets are lined up. The magnetic field cannot increase further. This is called **saturation**.

Some plug based supplies do not use a transformer at all. The rectify the mains voltage and the DC is passed through a **buck regulator** to supply the required voltage. One example is the kind of power supply to recharge a mobile phone.

Hysteresis, please, NOT ***hysteria***!

Transformers can only work with **alternating current**; they cannot work with direct current. This is because the magnetic field has to change for there to be a secondary voltage. If you put DC through a primary of a transformer, there is certainly a magnetic field in the core. It will be a lot more powerful than if there were an alternating magnetic field, but it would be steady, so there would be no secondary voltage. However, when the transformer is turned off, there would be a voltage across both the primary and secondary. If you happen to be touching a terminal at the time, it could give you a shock.

Old-fashioned car ignition systems used the repeated turning on and off if the 12 V battery voltage to induce a 25 kV voltage for the spark plugs in a **coil** (a step-up transformer). The switching on and off was done by a switch (the **points**) in the **distributor**. Nowadays, it is done electronically, and each spark plug has its own coil.

If an electric locomotive passes from 25 000 V ac to 3000 V dc overhead power line (as they do when passing from France to Belgium), a different solution is needed to step down the voltage. The 1500 V motors are put in series.

In the exam you may well be asked to discuss the causes of inefficiency in a transformer. Question 10.09.3 asks you to do that.

Tutorial 11.09 Questions

11.09.1

A power station generator generates 500 MW at a voltage of 15 000 V. Assuming it to be single phase.

(a) What is the current?

(b) The voltage is stepped up to 275 000 volts. Assuming that the transformer is 100 % efficient, what is the current in the secondary?

(c) What is the turns ratio in the transformer?

11.09.2

A transformer has a primary of 3600 turns and a secondary of 150 turns. It takes 1.5 amps from the 240 V mains.

(a) What is the turns ratio?

(b) What is the output voltage and current?

11.09.3

Transformers are very efficient machines but are not 100 % efficient. The best efficiency for a large power station transformer is about 97 %. Outline the reasons for this and discuss whether there is a limit to the power input and output of such a transformer.

11.09.4

A large industrial transformer steps an input voltage of 132 kV to an output voltage of 1000 V to provide power to an electric arc furnace. The electric arc furnace takes a current of 40 000 amps. The transformer is 95 % efficient. Assume the current is single phase.

(a) Calculate the power taken by the furnace.

(b) Calculate the input current.

(c) Calculate the power lost when the furnace is running.

(d) Suggest how the lost power is removed from the transformer to avoid damage due to overheating.

Tutorial 11.10 Transmission of Electricity

All Syllabi

Contents

11.101 Reducing Energy Loss

To carry the kinds of currents you get in a power station, you would need very thick wires. Here is the 3-phase output of a power station alternator (*Figure 109*).

Figure 109 Output wires of a power station alternator

Heavy currents make even thick wires get hot. The power lost is worked out using:

$$P = I^2R \quad \text{……………… Equation 85}$$

These high currents require very massive cables, which are cooled by oil. Question 11.10.2 asks you to consider the power losses.

These cables are about 30 metres long. Clearly the kinds of energy losses are completely unacceptable. At this rate, a line a few kilometres long will dissipate all the energy and would hardly light a torch bulb.

Your answer to 11.10.3 would be about the whole output of a big **power station**. That energy would be lost as heat, simply to warm up the countryside. So electrical energy is distributed at very high voltages, and relatively low currents.

You can see from your answer to 11.10.4 that the lost power is much less.

The output from a power station goes through a step-up transformer (*Figure 110*).

Figure 110 Step -up transformer (yes, we've seen it before)

11.102 The National Grid

In **power stations**, alternating current is generated typically at 25 000 V (with a current of 10 000 A) from each **alternator** (ac generator) per phase. The alternators are connected by short and massive cables to a **step up transformer** immediately outside the building. The voltage is stepped up to 275 kV. Much thinner cables can carry the electricity to where it's needed, using a network of high voltage **transmission lines** called the **National Grid**. The transmission lines are carried by **transmission towers** (pylons) to **substations** where the voltage is reduced by **step-down transformers**:

- 33000 V for local distribution.
- 25000 V for railways.
- 11000 V for heavy industry.
- 415 V for light industry.
- 230 V for our homes.

While such high voltages are potentially extremely dangerous, the distribution from large power stations is much more efficient that lots of small local power stations, so less fuel needs to be burned overall.

When working out energy losses, we can model the wires as perfect wires in series with a resistance (just like we modelled the cell with an internal resistance as a perfect battery in series with an internal resistor). This is shown in *Figure 111*.

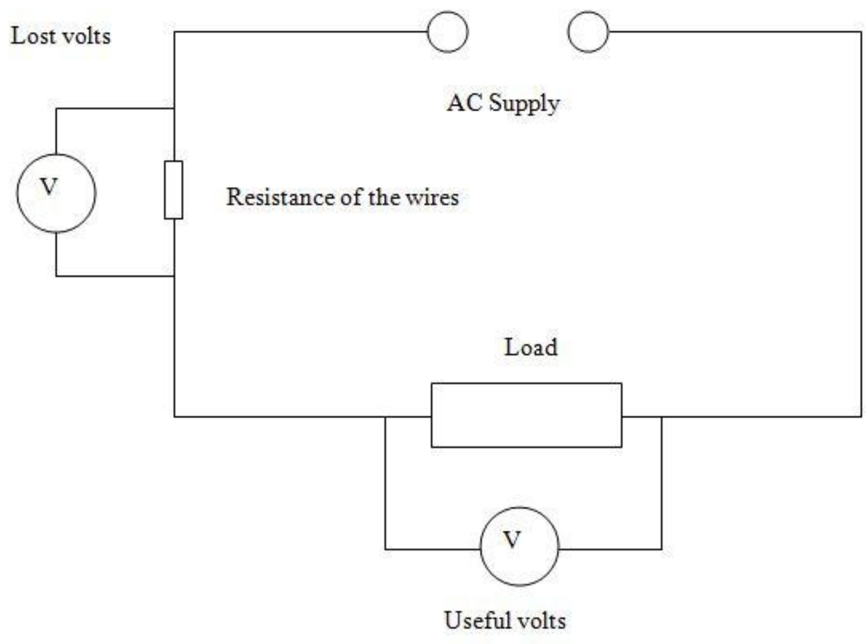


Figure 111 Modelling energy losses

The heavier the current, the greater the lost volts, and the lower the useful voltage to the load. This is illustrated in Question 11.10. 6. This question is challenging.

The National Grid was started in the late 1920s, and the design for transmission towers has not changed a great deal since then.

Our National Grid is connected to that of France by cables that run under the English Channel. These cables carry 2000 MW of electricity at 270 kV for a distance of 73 km. You may be surprised to note that this is **direct current**, not AC. The reason for this is that underground cables are not very efficient when they carry AC, due to energy losses as a result of capacitance. A coaxial cable makes a perfectly good **capacitor**. Its value may be low per metre, but when the cables run for several tens of kilometres, the capacitance becomes significant. The effect of capacitance is insignificant when a dc voltage is applied.

Converting AC to DC is easy; you use a **diode rectifier bridge**. On this scale it's big, but the concept is easy.

Converting DC to AC is not so easy; you need an **inverter**. With modern electronics, suitable devices can be made. Inverters are now available in the shops to power a mains device from a car battery.

11.103 Electric Cars (to think about)

Electric cars are nothing new. The example below (*Figure 112*) was built by Andreas Flocken (1845 - 1913) in Coburg. Built in 1888, it is one of the oldest cars in Germany.

Figure 112 An electric car from 1888 (Image by Henrtsirhenry, Wikimedia Commons)

It is a horse-drawn carriage from which the horse has been removed (and put out to pasture). A large lead-acid battery powers an electric motor that drives the rear wheels. It was by no means unique and in the late Nineteenth Century, there were many such vehicles on the road. They didn't need horses (which are expensive to maintain) and were clean and quiet. The problem was that they did not go very far, and finding a place to charge the battery was not always easy. The batteries were heavy.

The internal combustion engine provided a good answer. It could be started easily, and the cars had a reasonably good range. As petrol engines improved, cars became a lot more popular and competition between manufacturers gave them incentives to improve cars. Cars up to quite recently are much better than they were fifty years ago. They are safer, more comfortable, more economical, and much more reliable. However there have been retrograde steps in some cars, such as “wet belts” where the timing belts are lubricated with the engine oil. This causes the rubber belts to crack up and shed small bits of rubber to clog up the oiling system, leading to potentially catastrophic engine failure. Replacement of such a failed engine is eye wateringly expensive.

The problem with the internal combustion engine is that it has many moving parts which wear out. There are a number of sources of pollution caused by cars, for example exhaust emissions. Global warming has become a major concern with governments. Petrol engines were a major culprit. Therefore, the diesel car became more popular. The fuel consumption of diesel cars is quite a bit lower (15 % or more) than the petrol equivalents. Unfortunately, they are dirty, spewing out a lot of carbon particulates and nitrogen oxides. Modern diesel cars have complex exhaust systems to reduce the pollutants, but not with total success. Several high profile car manufactures have been caught out fiddling the emission results, so that the emissions of cars on test are very much less than the emissions while on the road. You can see that for yourself as you walk into your school or college. These systems are not always reliable and can be expensive to fix.

Now petrol cars are coming back into favour. However, they still perform badly in stop-start driving, the kind you get in cities. Many now have systems whereby the engine stops while waiting in a queue. It starts again when the car is ready to move off. (My car had this.) However, the repeated use of the starter motor (which takes a huge current, about 200 - 500 amps) will wear the component out and cane the battery. Both are expensive.

A solution to this has been the hybrid car, which uses an internal combustion engine in conjunction with an electric motor and a battery. However, they tend to be rather expensive. In the early part of the twenty-first century a Mayor of London ordered the introduction of diesel hybrid buses. These vehicles were driven by electric traction motors using very large batteries that were kept charged by a 4 litre diesel engine. Unfortunately, the batteries tended to fail, and there are piles of them behind the bus garages. The vehicles are now driven by their diesel engine using an electric transmission. They are heavy vehicles, and the resulting performance is rather poor.

Electric cars have also come back. They are particularly good for short distance motoring in stop-start traffic. They are quiet, comfortable, and reliable. They have lively performance. When going downhill they can generate electricity to charge up the battery. But despite improvements in battery technology, the batteries are bulky and have a limited range. For example, a Nissan Leaf has a range of 220 km, but this is reduced to 100 km if the heater and other accessories are on. Many owners become very anxious about the range of their EV.

This car is a Renault Zoe, a popular electric super-mini (*Figure 113*).

Figure 113 A modern electric car (Image by Vauxford, Wikimedia Commons)

The power of the motor is 66 kW (90 PS). It has a battery of capacity 41 kWh.

One electric supercar has a motor that can give out 800 kW (1200 PS). You can imagine that when you put your foot to the floor, the battery would go flat almost at once!

Most electric cars have lithium batteries. They are much lighter than lead acid batteries. They have large capacities. However, the disadvantages are that they are expensive. Also, they can catch fire if the casing is punctured (in an accident), or if the current demand is too high. Some people who bought electric cars a few years ago had to hire the batteries which added considerably to the monthly cost of electric motoring.

Most cars are highly computerised nowadays. Failures involving the computers seem to be common and expensive.

Electric cars are charged at home. It is possible to get chargers that plug into a standard 13 A socket, but these take a long time to charge up.

Larger chargers are available, but these need to be wired in to the house wiring. They take about 30 A, about the same as an electric shower. Suppose you have the electric shower on, the car charger on, the electric cooker, and a couple of electric heaters. (Possible for a small family.) The current will rapidly add up to above 100 A, quite sufficient to blow the board's fuse at the meter.

Then consider that, if every house had a charger, a considerable current demand would occur. This may have significant implications if the electrical infrastructure is old and/or in poor condition, as it is in many places. A government that chickens out at the cost of providing a couple of thousand gantries for the electrification of the railway between Kettering and Sheffield is hardly likely to take on the task of providing infrastructure for every house to accommodate a charger for their electric cars. And that isn't even considering the facilities needed for electric lorries or buses.

This may sound rather negative, and I would say that here is much genuine effort to improve electric vehicles, including improving the range, improving the cooling of batteries and enhancing the efficiency of electric motors. However, the ingenuity of scientists and engineers often gets pitted against the accountants who have to manage the eyewatering costs of developing a new car.

The extra loads have considerable implications for the National Grid, much of which was erected in the nineteen twenties and thirties.

And that doesn't start to address what would happen if, as is the case in many of our crowded cities, you find you can't park outside your house...

It is proposed to ban internal combustion vehicles by 2030. I will be well past my sell-by date then and will have progressed to the status of old codger. But you, dear reader...

Tutorial 11.10 Questions

11.10.1

This particular installation shown in *Figure* 107 generates at 15 000 V. Each alternator has a power output of 200 MW. What is the current in each of the three cables?

11.10.2

Each cable has a resistance of 0.001 Ω. How much energy is lost per second in each of the cables? How much is lost in total?

11.10.3

A low voltage transmission line is carrying a current of 30 000 A. Over its whole length, the resistance of the transmission line is 1.5 Ω. How much power is lost?

11.10.4

A high voltage transmission line is carrying a current of 1000 A. Over its whole length, the resistance of the transmission line is 1.5 Ω. How much power is lost?

11.10.5

The input voltage is 15 kV, and the output voltage is 275 kV. A step up transformer is transmitting 500 MW of power. Assume that the input and output are single phase.

(a) What is the input current?

(b) What is the output current?

(c) What is the turns ratio of this transformer?

11.10.6 (Challenge)

A farmer has an outlying barn 800 metres from his farm buildings. He wants a 230 V power supply to power a machine that takes 3 kW. He has the work done by a contractor who does the job on the cheap. He uses domestic cable that has a resistance of 0.045 ohms per metre. He buries the flex in a narrow trench that he digs across the fields with a pickaxe and spade.

(a) What is the total resistance of the cable to the barn?

(b) What is the resistance of the machine?

(c) What is the current used? (Hint: take into account the resistance of the wires)

(d) What is the voltage across the machine? Comment on the effect this would have on the performance of the machine.

(e) The farmer is not very pleased and gets another contractor who does the job properly. Discuss what the new contractor should do.

11.10.7

An electric car has a motor of power 66 kW (90 PS). It has a battery of capacity 41 kWh.

The car has a maximum speed of 140 km h^{-1}. Calculate the range of the car if it is driven at this speed.

11.10.8

Calculate how long it would take to charge a battery of capacity 41 kWh if the charger took 3 kW off the mains.

4. Reactive Components

Tutorial 11.11 Capacitors and Alternating Currents

SQA Advanced Higher and WJEC

Contents

The content of this tutorial is not on the AQA syllabus (nor, for that matter, the OCR or EDEXCEL). It is on the SQA Advanced Higher syllabus. It is part of the option A (Alternating Currents) on the Welsh Board (WJEC).

Before you study these notes, you should be familiar with the basic concepts of capacitors.

11.111 Phase in resistive AC circuits

Up to now, we have studied **resistive** components. If we connect a resistor across an alternating current, we see a sinusoidal graph like this (*Figure 114*).

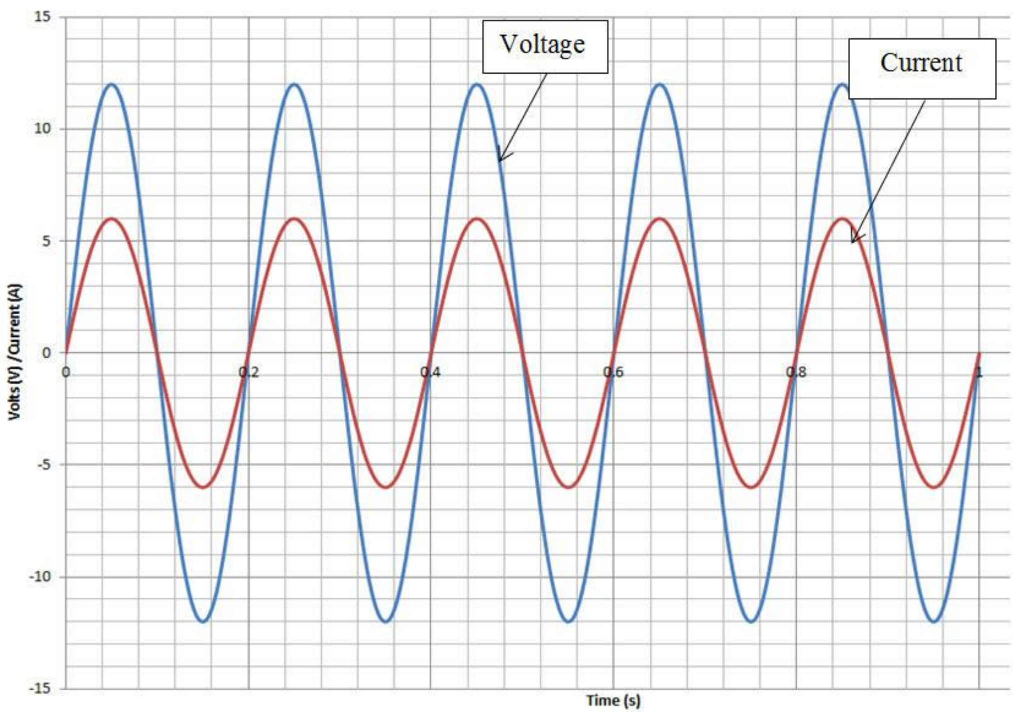


Figure 114 Current and Voltage in phase across a resistor

We notice that both the current and the voltage are in step, or **in phase**. We know that resistance is the opposition to the flow of current, and the resistance in this circuit remains constant whatever the frequency.

You will be familiar with the idea of **phase** from your study of simple harmonic motion. You may well have been told or worked it out for yourself that SHM and circular motion are inextricably linked. They use the same terms such as:

- angular velocity (ω).
- displacement (x).
- time (t).

Alternating current is generated by rotating machines. The voltage is linked with the rotation using the relationship:

$$E = BAN\omega \sin \omega t$$

.... *Equation 86*

11.112 Phasors

Have a look at this graph in which the current and voltage are 90° ($\pi/2$ rad) out of phase (*Figure 115*).

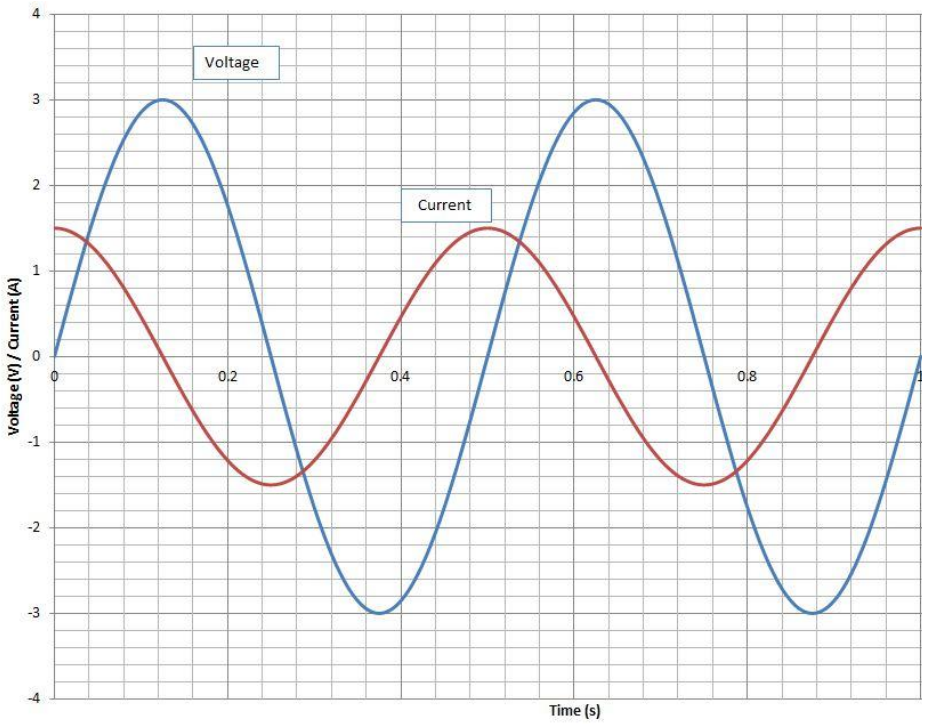


Figure 115 Voltage and current 90 degrees out of phase

Analysing a graph like this can be quite tedious. It is easier to use phasors. Quantities in alternating currents can be represented by **phase vectors** or **phasors**. A phasor is representation of a sinusoidal wave form as a **rotating vector**. Phasors are particularly useful when you have two or more alternating electrical quantities that are a fixed amount out of phase. This happens when we put **reactive** components like capacitors and inductors into an electrical circuit.

In our study of phasors, we will assume that:

- The **amplitude**, A, remains constant.
- The angular velocity, ω, remains constant.
- The phase relationship, ϕ, remains constant.

(The curious looking symbol, ϕ, is "phi", a Greek lower-case letter 'f' or 'ph'. It is the physics code for **phase angle**.)

Remember from circular motion and SHM that:

$$\omega = 2\pi f$$ .... *Equation 87*

(Note that some syllabuses use ω rather than $2\pi f$).

Let's look at a phasor that represents a sinusoidal waveform. It has frequency f, and an angular velocity of ω rad s^{-1}.

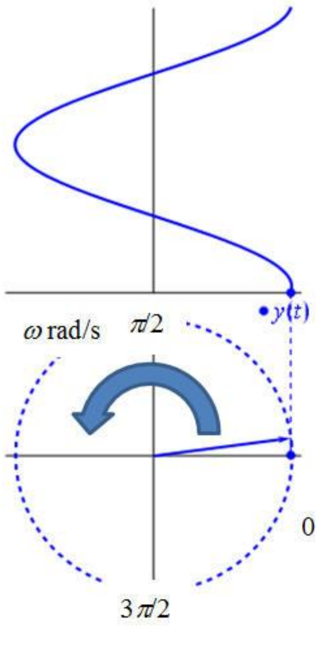


Figure 116 Phasor diagram

This diagram is showing how there is a rotating vector that is projected onto a moving piece of paper. It traces a sine wave.

The rotating vector is turning at a constant **angular velocit**y of ω rad s^{-1}. By convention it turns **anticlockwis**e. By convention, the zero point is the **3 o'clock position**. Angles are measured in **radians**.

In this tutorial we will consider the phase vectors are 90° (π/2 radians) apart.

For a resistive circuit, the phase vectors (phasors) are like this (*Figure 117*).

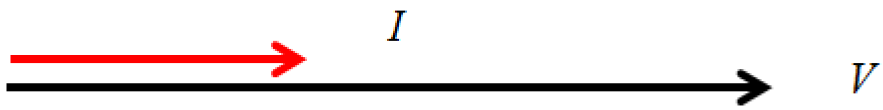


Figure 117 Phase vectors that are in phase

Note that for clarity they are shown parallel. Strictly speaking they should lie on top of each other.

11.113 Reactance

In AC circuits, there is another kind of opposition to the flow of current, called **reactance**. Reactance is defined as:

The ratio of the alternating voltage to alternating current.

$$X = \frac{V}{I}$$ .... *Equation 88*

The physics code for reactance is X and the units ohm (Ω).

Do NOT call reactance resistance!

It is different to resistance in that the voltage vector and the current vector are 90° apart.

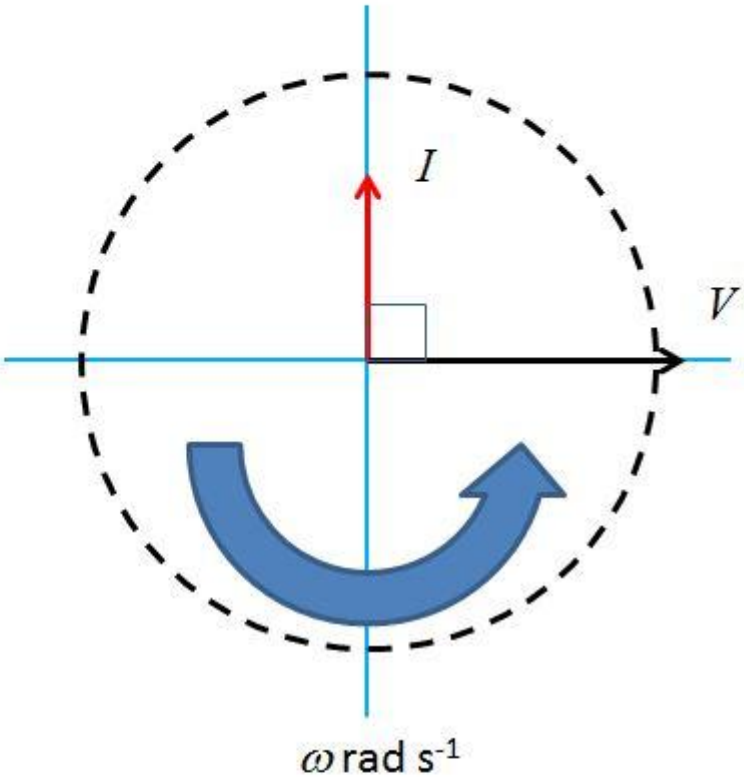


Figure 118 Current and voltage vectors 90 degrees apart.

There are two main components that have a reactance, which is frequency dependent. The reactance in a **capacitor** is the opposition to the change in voltage across the capacitor. The reactance in an **inductor** is the opposition to the change in current through the inductor. There is no reactance in purely resistive components.

Reactance is dependent on the **frequency**.

To make sense of reactance, we need to know something about the capacitor and the inductor. We will study the latter in Tutorial 11.12.

11.114 Reactance of a capacitor

Suppose we connect a capacitor in series with a light bulb. If we connect it to a DC supply, the bulb will (if the capacitor is large enough) momentarily flash and go out. If we turn off the supply and short out the terminals, the bulb will flash momentarily and then go out. We know that the capacitor has stored charge, and the charge passing through the bulb makes the filament glow for a very short time.

Now we set this circuit up with an **alternating** source (*Figure 119*):

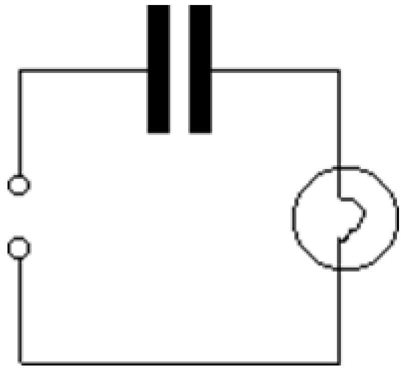


Figure 119 Capacitor in an AC circuit

If we connect a capacitor in series with a bulb:

- If connected to a low frequency, the bulb doesn't come on.
- If the frequency is increased, the filament starts to glow. It gets brighter as the frequency is increased.

The capacitor seems to block DC but allows AC to "flow". It's as if the bulb has some kind of resistance to AC. We say that the capacitor has a **reactance**. The resistance of a capacitor is **infinite** because there is a layer of insulating material that prevents current from passing between the plates. The reason that the current appears to flow is because the capacitor is constantly charging and discharging.

The higher the frequency the bigger the current that flows. This is because current is the **rate of flow of charge**.

$$I = \frac{dQ}{dt}$$

.... Equation 89

The capacitor does NOT conduct electricity.

Current does NOT flow through the capacitor.

The "flow" of a.c. is due to the charge and discharge of the capacitor.

If the capacitor conducts DC, it has failed.

From the statement above, we can say that the **reactance** decreases with frequency. Reactance is formally defined as:

the ratio of the potential difference to the current in a capacitor circuit.

In a capacitor circuit, the equation is written as:

$$X_C = \frac{V}{I}$$

.... Equation 90

Reactance is measured in ohms (Ω). The X_C term means that the reactance is that of a capacitor.

We can use this circuit to measure the voltage and current as we change the frequency in a capacitor circuit (*Figure 120*):

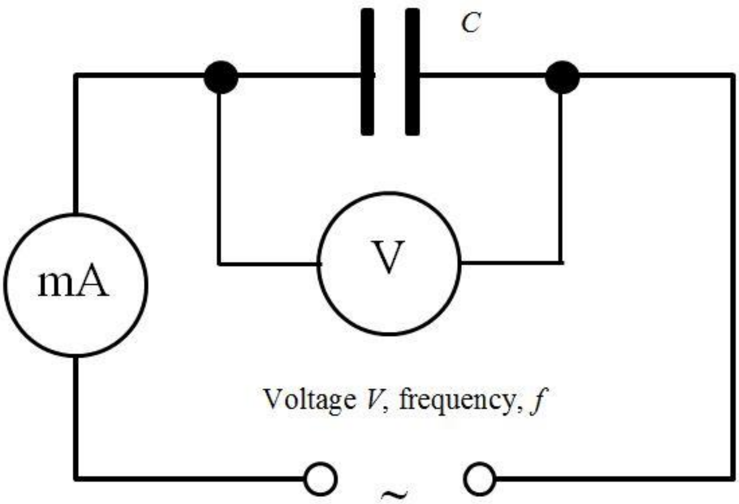


Figure 120 Changing the frequency and measuring voltage and current in a capacitor circuit

We can process the data to show us how reactance varies with frequency (*Figure 121*).

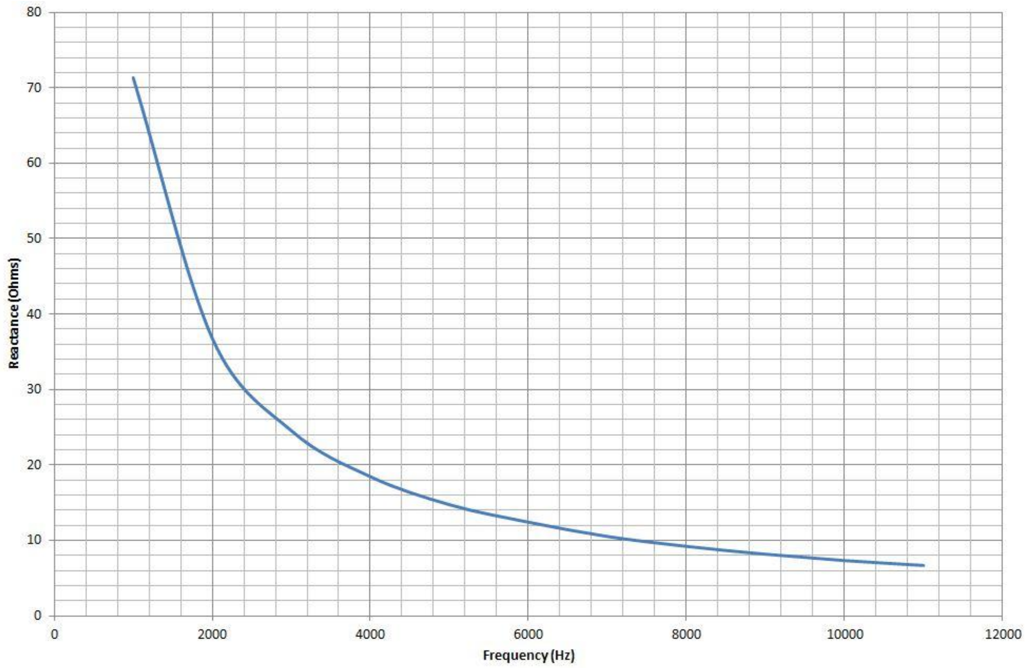


Figure 121 Graph of reactance against frequency

If we plot reactance against the reciprocal of frequency, we get a straight line (*Figure 122*).

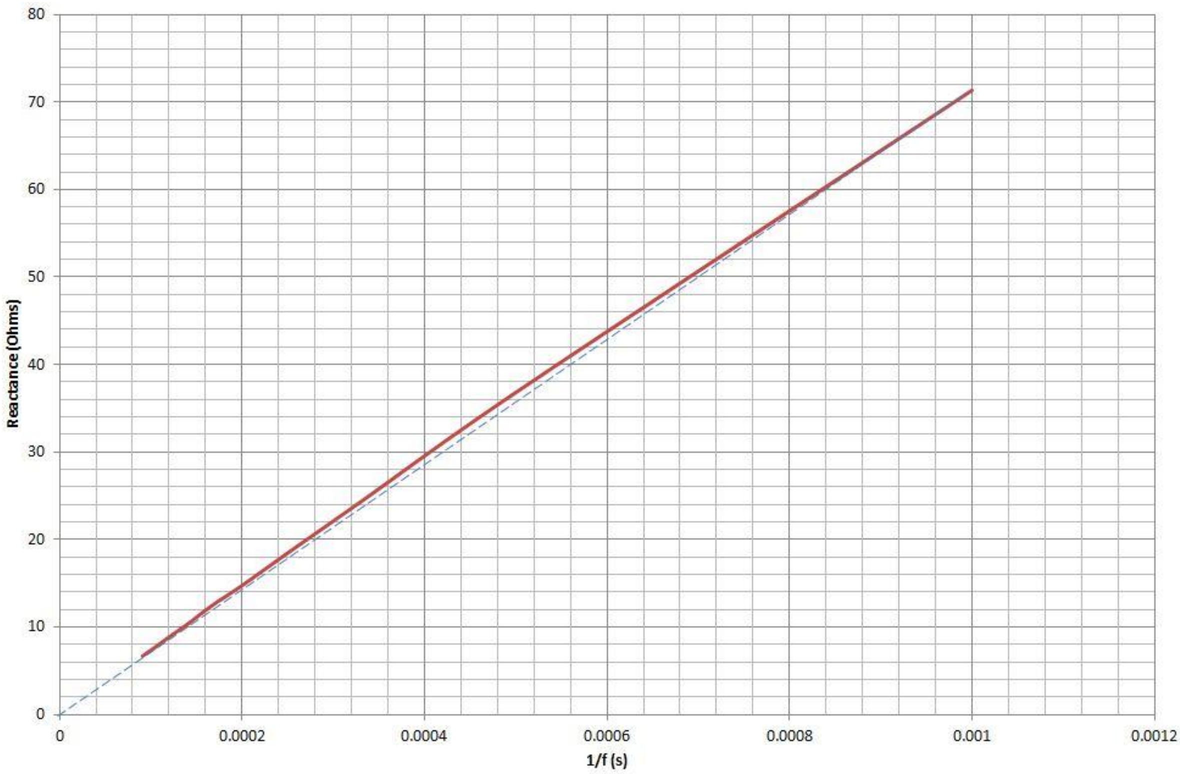


Figure 122 Graph of reactance against 1/frequency

These data were processed from an experiment, so the straight line is not quite perfect. The dotted blue line is the **line of best fit** and **extrapolated**. This tells us that the reactance is **inversely proportional to the frequency**.

For a capacitor of capacitance C connected to a voltage source with a frequency f, the reactance, X_C is given by:

$$X_C = \frac{1}{2\pi f C}$$

.... *Equation 91*

Worked Example
What is the reactance of a 47 μF capacitor connected to a 12 V AC supply that has a frequency of 500 Hz?
Answer Use: $X_C = \frac{1}{2\pi f C}$ $X_C = \frac{1}{2 \times \pi \times 500\ \text{Hz} \times 47 \times 10^{-6}\ \text{F}}$ $X_C = 6.8\ \Omega$

11.115 Simple CR Circuits

A pure capacitor connected to an alternating source is simply an electrical curiosity. In reality there are resistive elements, such as the internal resistance of the source, and the resistance of the wires. With resistive elements in the circuit, it becomes more interesting.

Let's use the same circuit as above, but this time we add a resistor, R (*Figure 123*).

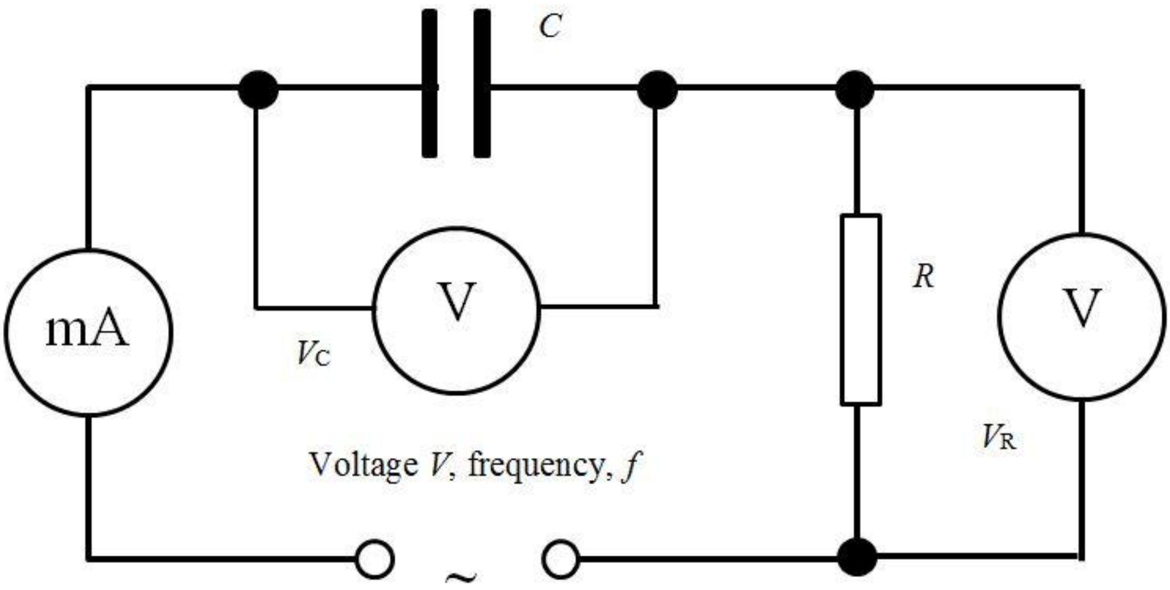


Figure 123 A simple CR circuit

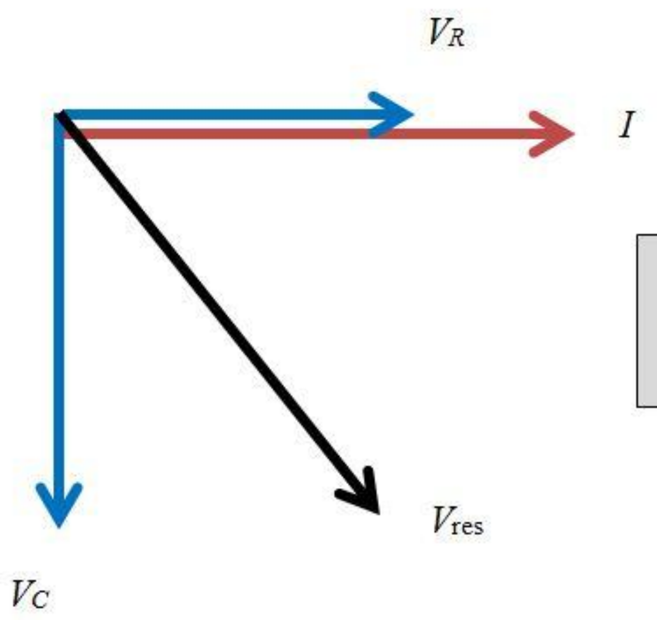
This is a simple series *CR* (or *RC*) circuit.

We will measure the voltage across the resistor as well as the capacitor.

We need to draw the current phasor first. By convention we always draw the quantity which is the same in a circuit first, i.e. at the zero position. The current into the capacitor leads the voltage by 90° (π/2 rad).

So, our current vector goes from left to right at the 3 o'clock position. Parallel to that is the **voltage across the resistor**.

The phasor diagram looks like this (*Figure 124*).



Remember **CIVIL**
Capacitor ***I*** leads ***V***.
V leads ***I*** for inductor, ***L***

Figure 124 Phasor diagram for the voltages in a CR circuit

The voltage across the capacitor and the resistor do NOT add up arithmetically.

If V_R = 3 V and V_C = 4 V:

$V_R + V_C$ ‡ 7 V

The voltage across the capacitor is at 90° and is lagging the voltage across the resistor, so its phase vector points vertically downwards. The resultant voltage is shown by the phasor V_{res}. We can work out V_{res} by simply using Pythagoras.

$$V_{res}^2 = V_R^2 + V_C^2$$

.... *Equation 92*

Since the voltage vector is pointing downwards, strictly speaking it should be negative. However, since the voltage is squared, the negative sign becomes positive, so don't worry about it.

11.116 Impedance

In a reactive circuit, we cannot talk about resistance as such.

So we have to introduce a new quantity, **impedance**, which is given the Physics code Z and has the units Ohms (Ω). Impedance takes into account the resistive and reactive elements in a circuit.

The formal definition of impedance is:

The ratio between resultant potential difference and the current in a reactive AC circuit.

We can write this as:

$$Z = \frac{V_{\text{res}}}{I}$$

.... *Equation 93*

We know that for the resistive elements:

$$R = \frac{V_R}{I}$$

.... *Equation 94*

We also know that for the reactance of a capacitor:

$$X_c = \frac{V_c}{I}$$

.... *Equation 95*

Since the current is the same, we can redraw our phasor diagram as (*Figure 125*):

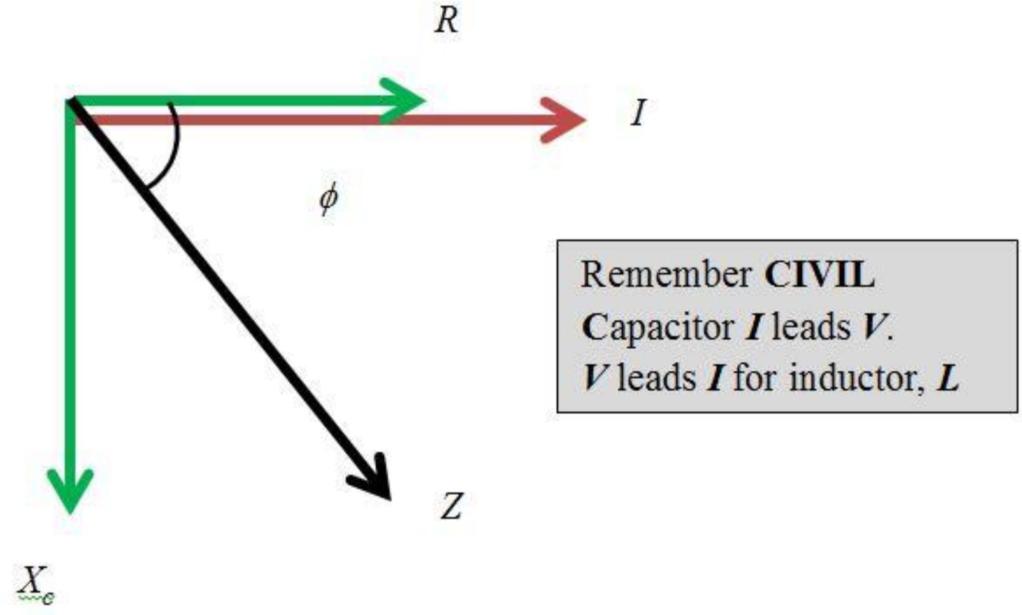


Figure 125 Phasor diagram for impedance

We can say that the impedance is the vector sum of the resistance and the reactance. So, by using Pythagoras again, we can write:

$$Z^2 = X_C^2 + R^2$$ .... *Equation 96*

In a **purely** capacitive circuit (i.e. with no resistor) the average power dissipated is zero. However, during each cycle, energy is transferred forwards and backwards.

Tutorial 11.11 Questions

11.11.1

A student writes defines the reactance of a capacitor as:

the ratio of the potential difference to the current flowing through a capacitor in a circuit.

Comment on this answer.

11.11.2

A 10 μF capacitor has a reactance of 320 Ω.
a) Show that the frequency of the AC supply is about 50 Hz;
b) Calculate the reactance at 1000 Hz.

11.11.3

What quantity, current or voltage, is always the same in a series circuit?

11.11.4

Explain why the two vectors are parallel.

11.11.5

What would be the resistance of a series RC circuit? Explain your answer.

11.11.6

At a certain frequency, a capacitor has a reactance of 20 ohms. It is in series with a resistor of 10 ohms. What is the impedance?

Tutorial 11.12 Inductors

SQA Advanced Higher and WJEC

Contents

The content of this tutorial is not on the AQA syllabus (or, for that matter, the OCR or EDEXCEL). It is on the SQA Advanced Higher syllabus. It is part of the option A (Alternating Currents) on the Welsh Board (WJEC). Other Option A content can be found in Tutorials 9 and 10.

I have included it here because students studying the AQA Electronics Option need to know something about the concepts of AC Theory to understand the ideas of filters. Also, students studying a syllabus that is not AQA will find these notes helpful.

11.121 Introducing Inductors

Before studying this tutorial, you may wish to revise Electromagnetic Induction (Tutorial 11.05).

Inductors are simpler components than capacitors. At its simplest an inductor is a coil of wire (*Figure 126*).

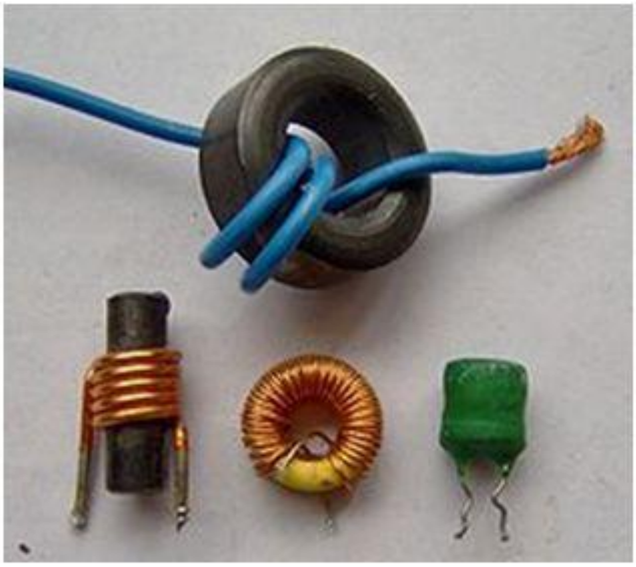


Figure 126 Inductors

In the picture above (*Figure 124*), the wire is wrapped around a magnetic **core**. The coils for a demountable transformer can be used as inductors (*Figure 127*).

Figure 127 Coils for a transformer are inductors

Capacitors are very sensitive to changes in temperature or exceeding the working voltage. There are no such problems with inductors.

There are different kinds of inductors as shown as symbols in the picture below (*Figure 128*):

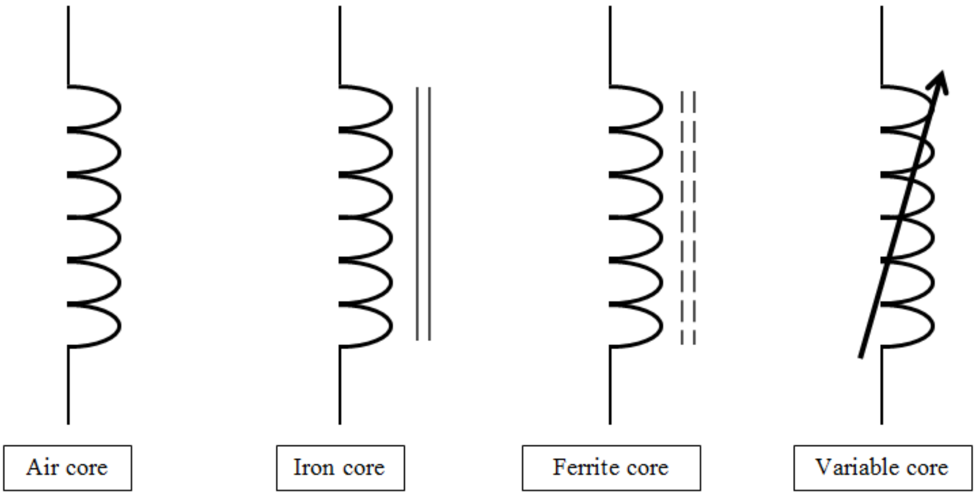


Figure 128 Symbols for various kinds of inductors

We will consider single inductors which have a self-inductance, for which the Physics code is L and the units are Henrys (H). The unit is named after Joseph Henry (1797 – 1878), an American physicist, who did a lot of pioneering work with electromagnetism.

As far as electric currents are concerned, an inductor is simply a piece of coiled wire and should not affect the flow of charge at all. A **perfect** inductor has **zero** resistance.

Suppose we connect an inductor in series with a bulb. The circuit is below (*Figure 129*):

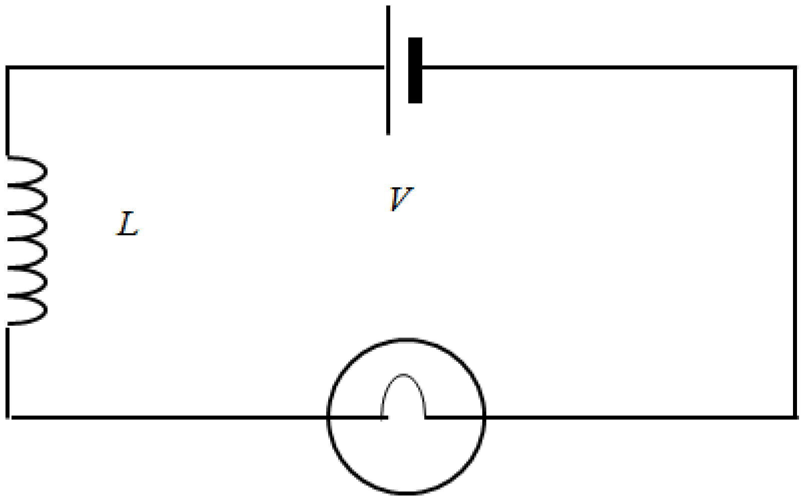


Figure 129 In inductor in a DC circuit in series with a bulb

The inductor has zero resistance. If we connect the circuit to a DC battery, the bulb lights up to full brightness, just as we would expect.

Now let's connect it to an AC supply of exactly the same voltage. We would expect the bulb to light up to exactly the same brightness.

But we see that the bulb is slightly dimmer. There seems to be some extra "resistance". This "resistance" is caused by the **reactance** of the inductor.

11.122 Why does an inductor have a reactance?

Electric currents always produce a **magnetic field**. That is an unchangeable law of Physics. A direct current produces a **steady** magnetic field, while an alternating current produces a magnetic field that is **changing** all the time.

You can make a voltage across the ends of a wire by **moving** a magnet past a wire, or by moving a wire past a magnet. You can even make a voltage by having a magnet sitting next to a wire that is stationary, but you have got to **change** the magnetic field. If the magnetic field remains the same, no voltage will be induced, however strong the magnetic field.

A current-carrying coil of wire will act as an **electromagnet**, even though the coil itself is made of a non-magnetic material like copper. Let's think about what would happen when we switch on a current that passes through a coil of wire. When we switch on the current, a magnetic field is made as the current flows. As the field being made, a **reverse** voltage is made to **oppose** the increase in voltage across the inductor.

We can use **Faraday's** and **Lenz's laws** to help us to model the situation. There are some terms that we need to know to help us think it through:

Quantity	***Physics Code***	***Units***
Flux density (magnetic field strength)	B	Tesla (T)
Magnetic flux	F	Weber (Wb)
Electromotive Force (voltage)	$\mathcal{E}$ (Curly 'E')	Volts (V)

You may remember the magnetic field of a bar magnet looking like this (*Figure 130*).

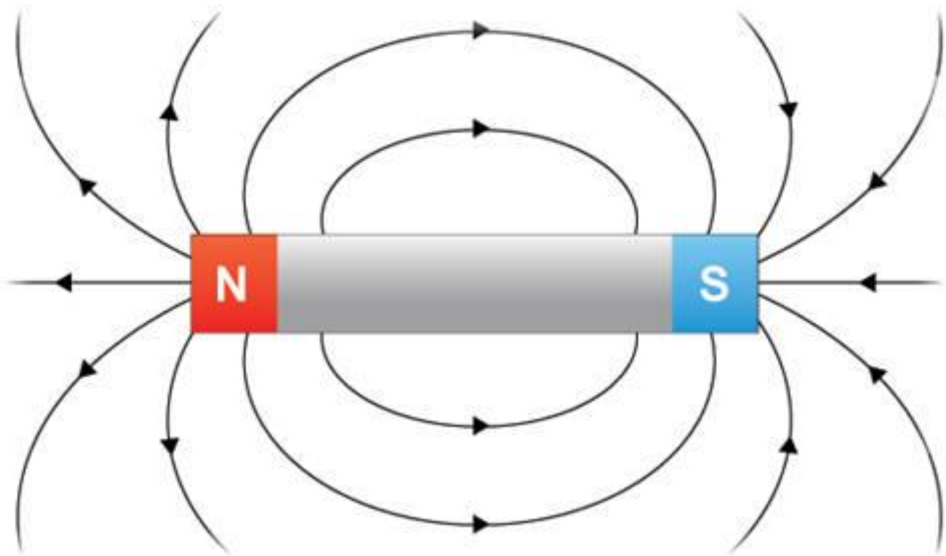


Figure 130 Flux lines around a bar magnet

Flux is simply the total number of field lines. There are 9 field lines in this picture.

Flux density is high when the field lines are close together. That means that the magnetic field is strong. So, the magnetic field strength is shown by the **concentration** of field lines. If the field line a spread out, the field is weaker.

The magnetic flux is the product between the field strength and the area. In physics code it is written:

$$\Phi = BA$$

.... *Equation 97*

Faraday's and Lenz's Laws tell us that a **change** in a magnetic field will produce an electromotive force (EMF) that will act to **oppose** the change. This can be summed up by the equation:

$$\mathcal{E} = -N\frac{d\Phi}{dt}$$

.... *Equation 98*

The term N is the number of turns in the coil, and the $d\Phi/dt$ is the **rate of change of flux**, i.e. how much the flux changes in a small time interval. If the dt term is very small, then the reverse EMF is very large. This has important implications for electronic circuits with inductive components.

If we plot a graph of the voltage across an inductor when it is switched on, we get something like this (*Figure 131*).

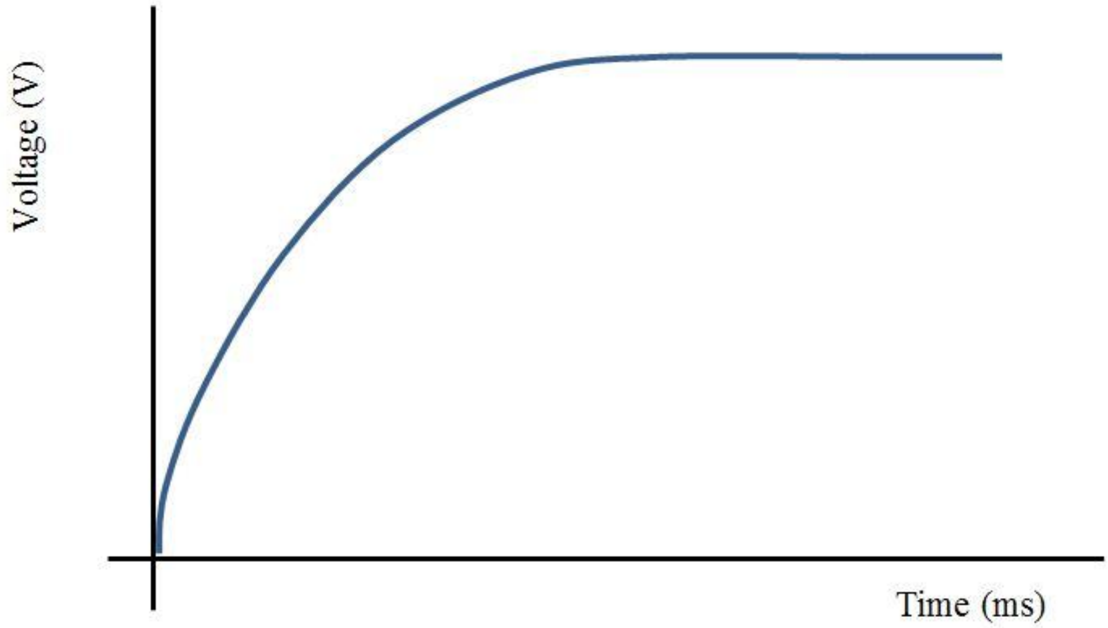


Figure 131 Inductive rise in an inductor

When the voltage is not changing when DC flows, then there is zero reverse EMF, and the current flows normally as if the inductor were simply a wire. However, some work per coulomb is done to build up the magnetic field when the inductor is switched on. The rise in voltage is **exponential**, but further study is not needed at this level.

However, in AC, as the current is changing direction all the time, there is a reverse EMF being induced all the time to prevent the change from happening. The higher the frequency, the bigger the reverse EMF.

11.123 Inductance

The inductance of an inductor is the property by which the inductor induces a voltage in itself in response to a **changing** current. It has the Physics code L and is measured in Henrys (H). Consider a simple inductor, which is a coil of wire wrapped around a cylindrical piece of core material, which might not even be magnetic (*Figure 132*).

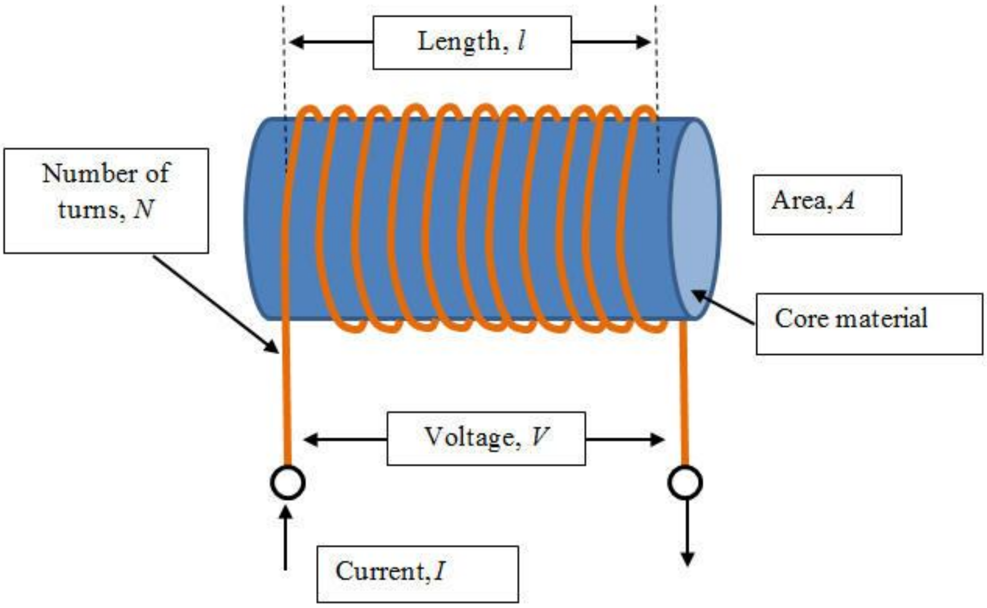


Figure 132 A simple inductor

The equation that links all these terms together is:

$$L = \frac{\mu_0 N^2 A}{l}$$

.... *Equation 99*

The terms involved are:

Quantity	***Physics Code***	***Units***
Inductance	L	Henry (H)
Permeability of free space	μ_0 $(= 4\pi \times 10^{-7})$	H m^{-1}
Number of turns	N	
Area of the solenoid	A	m^2
Length	l	m

The **permeability of free space** is a constant that is common in electromagnetism. Its Physics code is μ_0, (pronounced "mu-nought"). The symbol μ is "mu", a Greek lower-case letter 'm'.

11.124 Inductance and Current

The self-inductance of a coil can be worked out from the equation:

$$\mathcal{E} = -L\frac{dI}{dt}$$

...............*Equation 100*

From this, we can say that the induced EMF is the product of the **inductance** and the **rate of change of the current**.

- The dI/dt term is the rate of change of the current.
- Units for dI/dt are amps per second (A s^{-1}).
- The minus sign tells us that the EMF is opposing the applied voltage.

The graph below shows the idea (*Figure 133*).

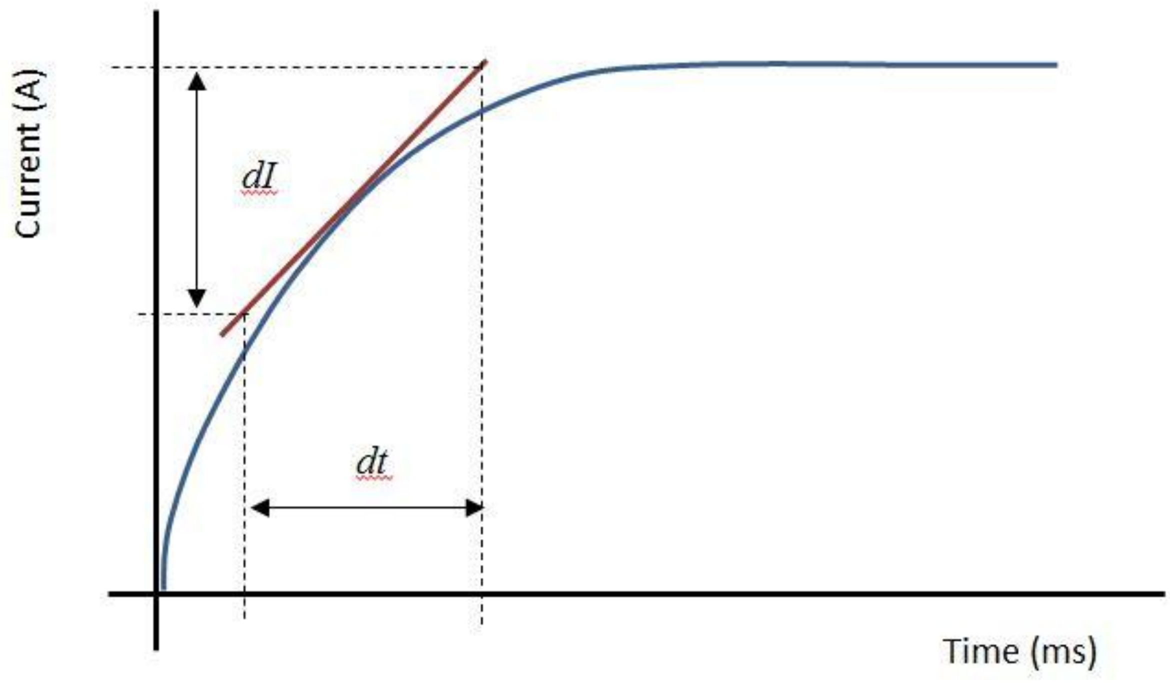


Figure 133 Showing the rate of change of current

The dI/dt term is obtained by taking the **tangent** from the graph and working out its rise and run. This is not a particularly easy way of measuring L. But if we know L, we can easily work out the reverse EMF. Notice that the maximum rate of change of the current is just when the inductor is turned on.

11.125 Energy in an Inductor

If we have an inductor in a DC circuit, we find that it makes little difference. However, if we turn the current off, the magnetic field collapses. A large reverse EMF is produced that can give you a shock as the energy is released.

The reverse voltage spike will wreck electronic components.

The energy held in an inductor is given by the equation:

$$E = \frac{1}{2}LI^2$$

.... *Equation 101*

Worked example What is the energy stored in a inductor of 0.5 H when a current of 4.0 amps is flowing through it?
Answer E = ½ × 0.5 H× 4. 0 (A)^2 = **4 J**

If it takes 0.1 s for the magnetic field to collapse, the 4.0 J is dissipated in 0.1 s, i.e. at a rate of 40 W.

Tutorial 11.12 Questions

11.12.1

Explain which AC voltage we should use - peak or RMS.

11.12.2

An inductor is made of a solenoid of 1200 turns of copper wire around a hollow square former 2.0 cm × 2.0 cm. The length of the solenoid is 5.0 cm. Calculate the inductance.

11.12.3

A solenoid of 2.3×10^{-2} H and negligible resistance is connected across a 12 V battery. What is the rate of increase of current in the solenoid as it's turned on?

11.12.4

A current of 5.0 A is flowing through an inductor of 10 H.
(a) What is the energy contained in the inductor?
(b) What is the reverse voltage if it takes 0.05 s for the magnetic field to collapse?

Tutorial 11.13 Inductors and Alternating Currents

SQA Advanced Higher and WJEC

Contents

Tutorial 12 - (A-level Extension)

The content of this tutorial is not on the AQA syllabus (nor, for that matter, the OCR or EDEXCEL). It is on the SQA Advanced Higher syllabus. It is part of the option A (Alternating Currents) on the Welsh Board (WJEC).

I have included it here because students studying the AQA Electronics Option need to know something about the concepts of AC Theory to understand the ideas of filters. Also, students studying a syllabus that is not AQA will find these notes helpful.

Before you study this tutorial, make sure that you understand inductors (Tutorial 11.12).

11.131 Reactance of an Inductor

Consider this circuit (*Figure 134*).

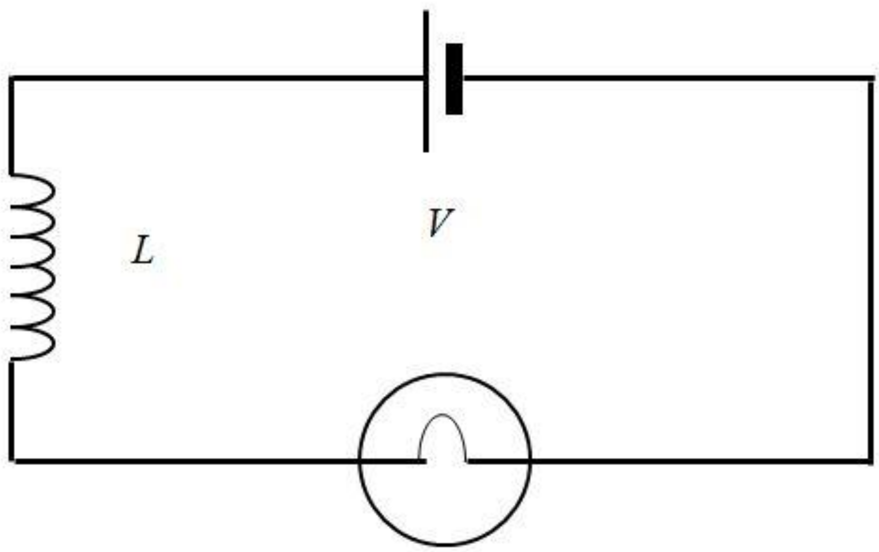


Figure 134 Inductor in series with a bulb in a DC circuit

When the bulb is connected to DC, it glows with full brightness.

When it is connected to a signal generator giving out the RMS voltage that corresponds with the DC voltage, we observe the following:

- At low frequencies, the bulb in series with the inductor is bright.
- As we increase the frequency, the bulb becomes dimmer.
- It allowed DC to flow freely, but opposed the flow of AC.

It is as if the inductor has a kind of resistance to AC, which is the **reactance**.

Electrically, an inductor is simply a wire coiled up in a former or round a core. A **perfect** inductor has **zero** resistance. In reality there is resistance, because copper wire has resistance, but it is very low. In this section we will assume that the inductor is perfect. The equation for reactance is:

$$X_L = 2\pi f L$$

.... *Equation 102*

Worked example What is the reactance of a 3.5 mH inductor connected to a 12 V AC supply that has a frequency of 500 Hz?
Answer: $X_L = 2\pi f L$ $X_L = 2\pi \times 500\text{ Hz} \times 3.5 \times 10^{-3}\text{ H}$ $X_L = 11\ \Omega$

If we plot **reactance** against **frequency**, the graph is a straight line of positive gradient going through the origin. This means that the reactance is **directly proportional** to the frequency for a perfect inductor. The graph is like this (*Figure 135*)

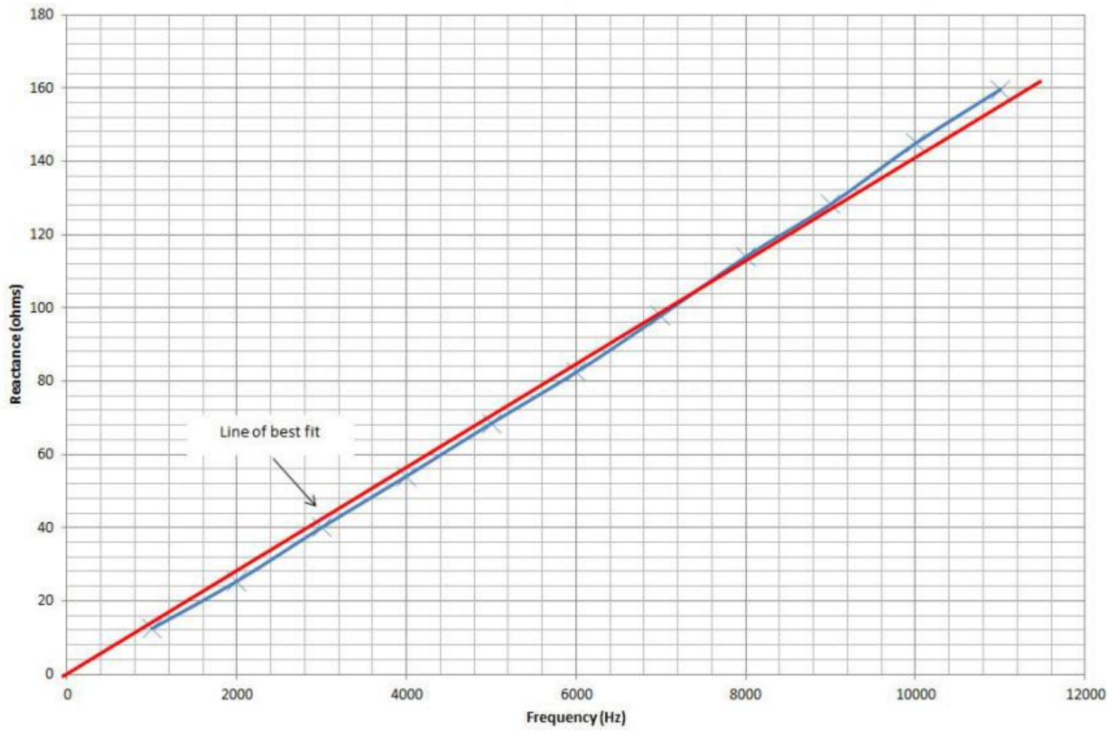


Figure 135 Graph of reactance against frequency for an inductor.

These data were taken from a real experiment, and a line of best fit was added. The points are slightly away from the ideal line predicted by the equation. This is because the inductor will have a certain resistance.

You can determine the value of the inductance by taking the **gradient** and dividing by 2π.

11.132 Phase relationship in an inductor

If we plot the current and voltage in a pure inductor against the time, we get a graph like this (*Figure 136*)

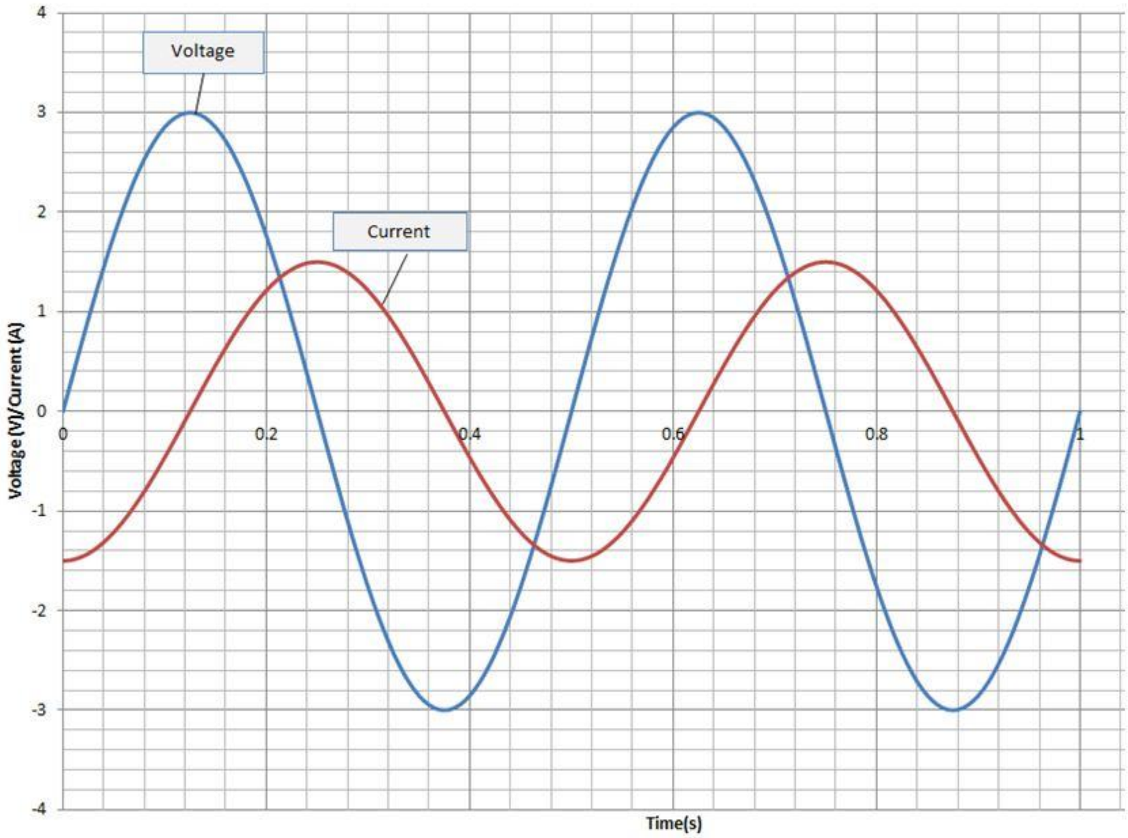


Figure 136 Phase relationship between voltage and current in an inductor.

The current graph is **lagging** the voltage graph by 90° or π/2 rad. So, the voltage phase vector is **leading** the current phasor by π/2 rad. We can show this in a phasor diagram as this (*Figure 137*).

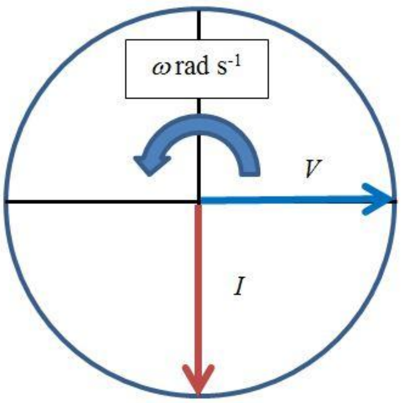


Figure 137 Phasor diagram for an inductor

Note that a *purely* reactive circuit based on a perfect inductor does not cause energy to be used. Energy is merely shuttled backwards and forwards between the inductor and the source. In reality there are resistive elements in any inductor circuit, not least in the inductor itself. Measuring current with a milliammeter introduces a resistive element to the circuit as well.

11.133 Simple *LR* Circuit

A purely inductive circuit is simply an electrical curiosity (and doesn't exist in reality). With resistive elements in the circuit, it becomes more interesting. There are resistive elements, such as the internal resistance of the inductor, and the resistance of the wires.

Consider this circuit (*Figure 138*).

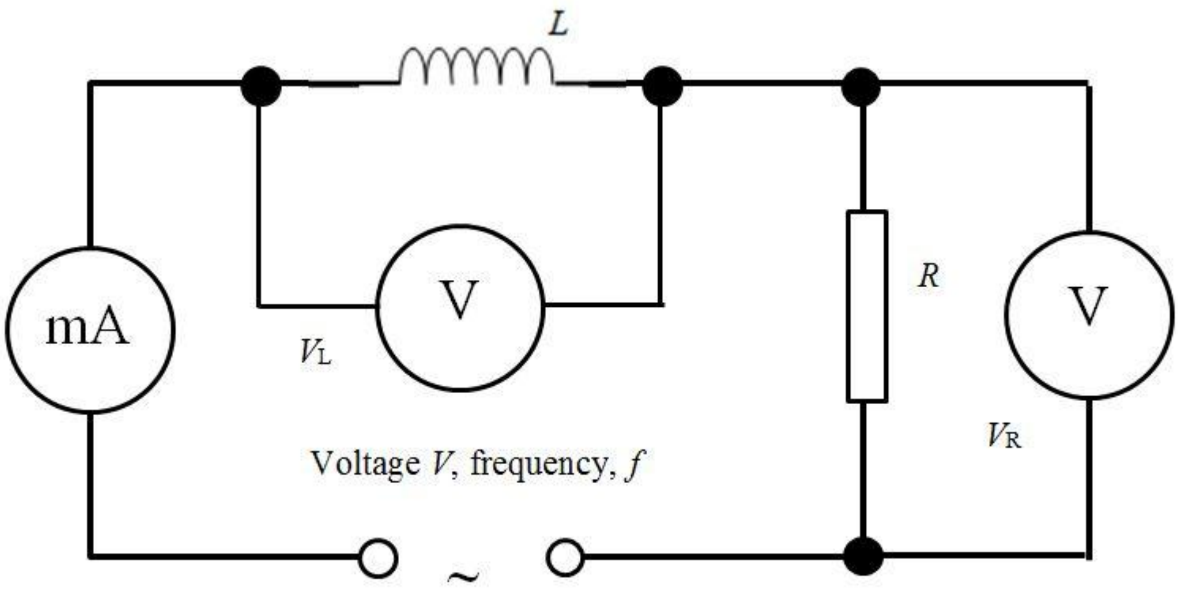


Figure 138 A simple LR circuit

This is a simple series LR circuit.

We will measure the voltage across the resistor as well as the inductor.

We need to draw the current phasor first. By convention we always draw the quantity which is the same in a circuit first, i.e. at the zero position (*Figure 139*).

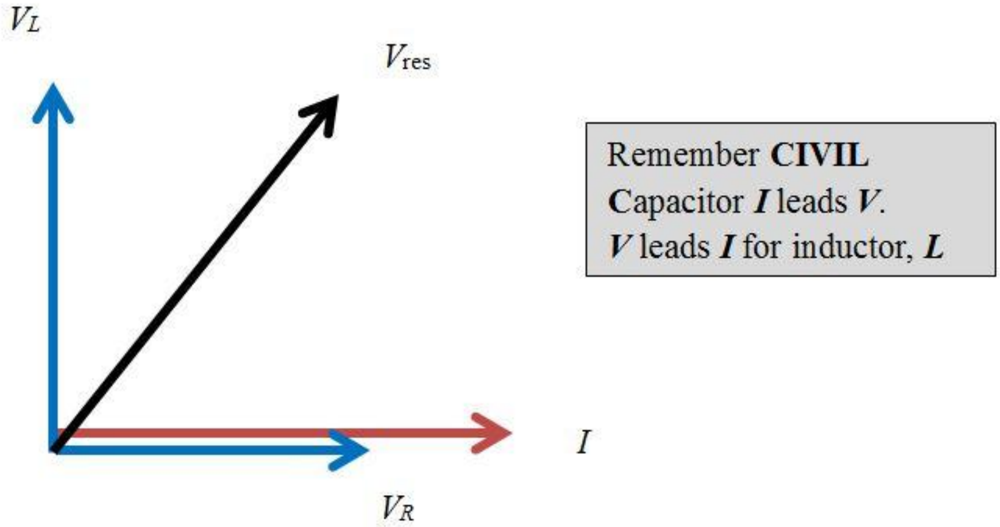


Figure 139 Voltage phase vectors in an inductor

The voltage across the inductor is at 90° and is leading the current, so its phase vector points vertically upwards. The resultant voltage is shown by the phasor V_{res}. We can work out V_{res} by simply using Pythagoras.

$$V_{res}^2 = V_R^2 + V_L^2$$

.... *Equation 103*

11.134 Impedance of the *LR* Circuit

When we studied capacitors, we introduced a new quantity, **impedance**, which was given the Physics code Z and had the units Ohms (Ω). Impedance takes into account the resistive and reactive elements in a circuit. The same applies to reactance of an inductor.

The formal definition of impedance is:

The ratio between resultant potential difference and the current in a reactive AC circuit

We can write this as:

$$Z = \frac{V_{\text{res}}}{I}$$

.... *Equation 104*

We know that for the resistive elements:

$$R = \frac{V_R}{I}$$

.... *Equation 105*

We also know that for the reactance of an inductor:

$$X_L = \frac{V_L}{I}$$

.... *Equation 106*

Since the current is the same, we can redraw our phasor diagram as:

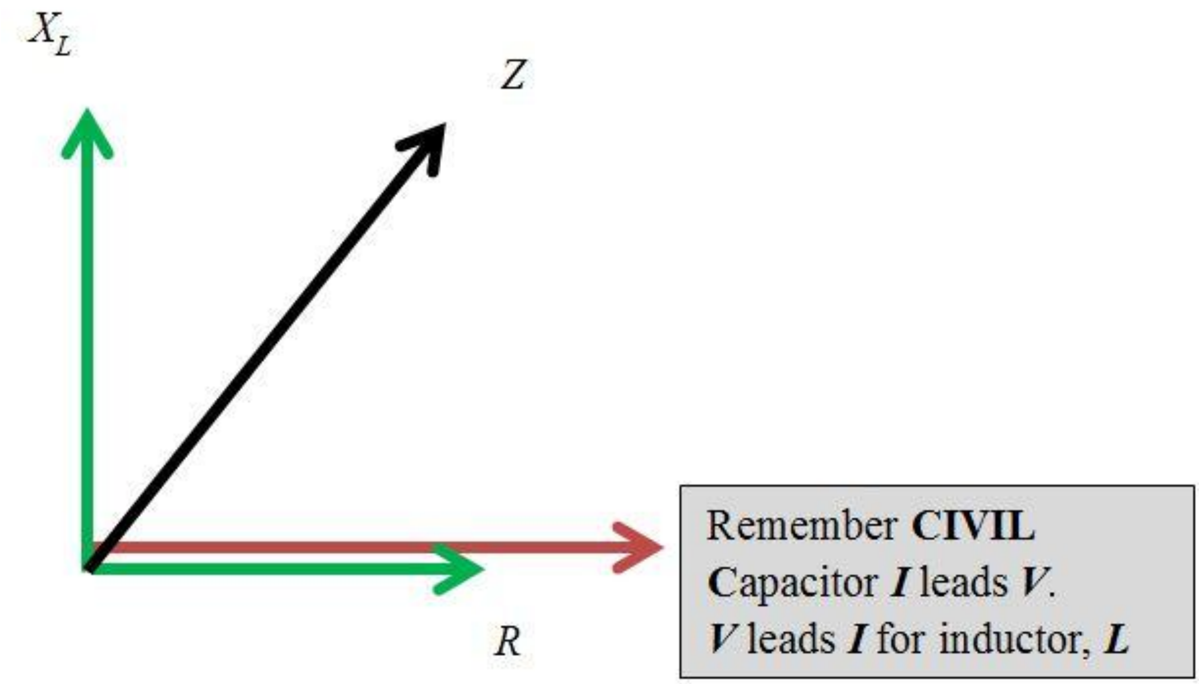


Figure 140 Phase vector diagram with resistance and reactance for an inductor.

We can say that the impedance is the **vector sum** of the resistance and the reactance. So, by using Pythagoras again, we can write:

$$Z^2 = X_L^2 + R^2$$

.... *Equation 107*

In a **purely** inductive circuit (i.e. with no resistor) the average power dissipated is zero. However, during each cycle, energy is transferred forwards and backwards.

Tutorial 11.13 Questions

11.13.1

A 10 mH inductor has a reactance of 320 Ω.
(a) Show that the frequency of the AC supply is about 5100 Hz;
(b) Calculate the reactance at 1000 Hz.

11.13.2

At a certain frequency, the voltage across an inductor is found to be 3.5 V while the voltage across the resistor is found to be 4.0 V.
(a) Why is the total voltage not 7.5 V?
(b) What is the resultant voltage?

11.13.3

At a certain frequency, an inductor has a reactance of 20 ohms. It is in series with a resistor of 10 ohms. What is the impedance?

11.13.4

A fluorescent tube for use in a 50 Hz mains light fitting is marked 240 V, 60 W. In operation it has a resistance of 50 ohms and has an inductor (sometimes called a choke) to limit the current.

(a) What power would the tube use if it were directly connected to the mains?

(b) Work out the current in a 50 ohm resistor if the correct power of 60 W is flowing through it.

(c) Show that the voltage across the resistor is about 55 V.

(d) Using a phasor diagram, or by calculation, calculate the voltage across the coil.

(e) Work out the reactance of the coil.

(f) Work out the inductance of the coil. Give your answer to an appropriate number of significant figures.

(g) Calculate the impedance of the light fitting.

(h) Comment whether the value you worked out in (g) is consistent with the resultant voltage of 240 V.

Answers to Questions

Tutorial 11.01

11.01.1

Use $F = BIl$

0.01 N = $B \times$ 2.5 A $\times$ 0.05 m

B = **0.080 T**

11.01.2

Use $F = BIl \sin\theta$

0.01 N = $B \times$ 2.5 A $\times$ 0.05 m × sin 35

B = 0.139 T = **0.14 T** (2 s.f.)

Tutorial 11.02

11.02.1

(a)

τ = 14 × 10^{-3} T × 15 × 10^{-4} m^2 × 0.25 A × 500 = 2.625 × 10^{-3} N m = **2.6 × 10^{-3} N m (2 s.f.)**

(b)

τ = 2.625 × 10^{-3} N m × cos 27° = 2.339 × 10^{-3} N m = **2.3 × 10^{-3} N m (2 s.f.)**

11.02.2

(a)

τ = 140 × 10^{-3} T × 27 × 10^{-4} m^2 × 2.0 A × 500 = 0.378 N m = **0.38 N** m (2 s.f.)

(b)

f = 3000 rpm ÷ 60 s = 50 Hz.

P = 0.378 N m × 2 × π × 50 Hz = 118.75 W = **120 W** (2 s.f.)

(c)

Total power = 118.75 W ÷ 0.80 = 148.44 W

V = 148.44 W ÷ 2.0 A = 74.22 V = **74 V** (2 s.f.)

Tutorial 11.03

11.03.1

$$F = Bqv$$

$= 0.82\ \text{T} \times 1.6 \times 10^{-19}\ \text{C} \times 6.0 \times 10^{6}\ \text{m s}^{-1}$

$F =$ **7.9 × 10^{-13} N**

The value of the force would be the same…

But the direction would be opposite.

11.03.2

$$F = mv^2/r \text{ and } F = Bqv$$

$$\Rightarrow mv^2/r = Bqv$$

$$\Rightarrow mv/r = Bq \Rightarrow m = Bqr/v$$

$$m = \frac{0.920\ \text{T} \times 1.60 \times 10^{-19}\ \text{C} \times 0.500\ \text{m}}{3.00 \times 10^{7}\ \text{m s}^{-1}}$$

$m =$ **2.45 × 10^{-27} kg** (3 s.f.)

This is 2700 times the mass of an electron.

11.03.3

$$E = \frac{V_H}{d}$$

$E = 0.35\ \text{V} \div 5.0 \times 10^{-3}\ \text{m} =$ **70 V m^{-1}**

11.03.4

Formula:

$$V_{\mathrm{H}} = \frac{BI}{ntq}$$

Charge carriers per unit volume for germanium is 2.02×10^{21} m^{-3}

V_{H} = (0.14 T × 0.067 A) ÷ (2.02×10^{21} m^{-3} × 0.35×10^{-3} m × 1.602×10^{-19} C)

= **8.3×10^{-2} V** = 82 mV

11.03.5

Use:

$$m = \frac{Bq}{2\pi f}$$

$$m = \frac{1.50\ \text{T} \times 1.60 \times 10^{-19}\ \text{C} \times 2}{2 \times \pi \times 2.00 \times 10^{6}\ \text{Hz}}$$

m = **3.82×10^{-26} kg** (3 s.f.)

Tutorial 11.04

11.04.1

Flux linkage = NBA

= 200 turns $\times$ 2.5 $\times$ 10^{-3} T $\times$ 0.1 m^2

= **0.05 Wb**

11.04.2

Original flux linkage = 0.05 Wb

New flux linkage = 200 $\times$ 2.5 $\times$ 10^{-3} T $\times$ 0.1 m^2 $\times$ sin 60

= 0.05 Wb $\times$ 0.866 = 0.0433 Wb

$\Delta\Phi$ = 0.0433 Wb – 0.05 Wb = **- 6.7 $\times$ 10^{-3} Wb**

Tutorial 11.05

11.05.1

$\mathcal{E} = -N\,\Delta\Phi/\Delta t$ and $\Delta\Phi = B\Delta A$

$$0.75\text{ V} = (-)\frac{2500 \times B \times (0 - 1.5 \times 10^{-4}\text{ m}^2)}{0.3\text{ s}}$$

$$B = \frac{0.75\text{ V} \times 0.3\text{ s}}{2500 \times 1.5 \times 10^{-4}\text{ m}^2}$$

B = **0.6 T**

11.05.2

(a)

The length, as this is taken into account in the speed.

(b)

$\mathcal{E} = NvwB$ = 30 × 1.20 × 0.08 × 0.245 = **0.71 V**

11.05.3

Use $\mathcal{E} = NvwB$

We need to work out v:

Length l of the beer passes every second.

The volume of the cylinder is $\pi r^2 l = 0.4\text{ m}^3$.

$\Rightarrow l = 0.4 \div (\pi \times 0.175^2) = 4.16\text{ m}$

$\Rightarrow v = 4.16\text{ m s}^{-1}$

Treat the liquid as a single turn:

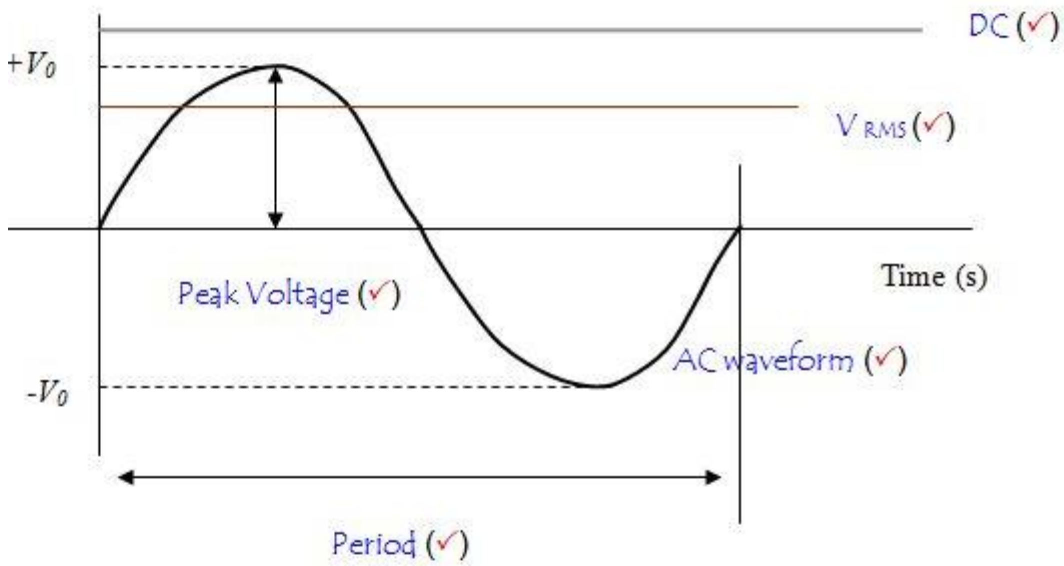
$\mathcal{E} = 4.16\text{ m s}^{-1} \times 0.35\text{ m} \times 5 \times 10^{-3}\text{ T}$ = **7.3×10^{-3} V** = 7.3 mV

Tutorial 11.06

11.06.1

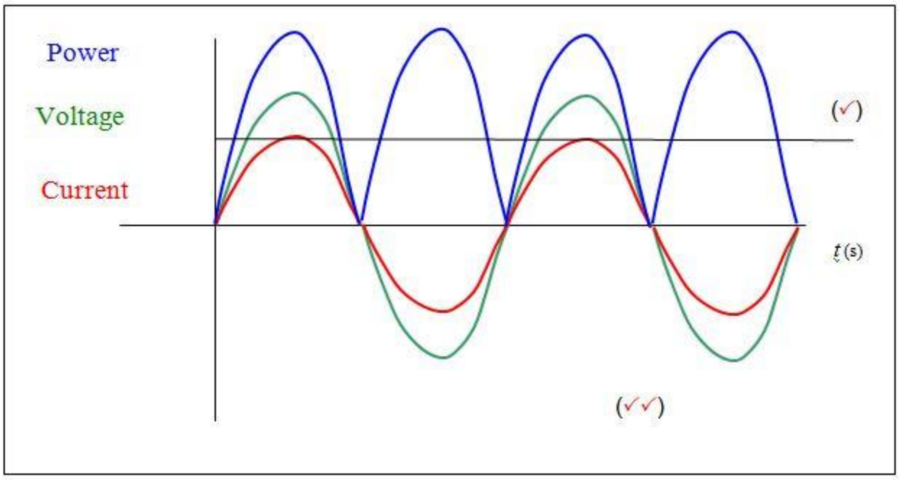
$$V_{pk} = 230 \times \sqrt{2} = \mathbf{325\ V}$$

11.06.2



11.06.3

Power is always positive.
(since both positive and negative values of voltage and current give a positive power).
It is in phase with the voltage and current.
Power varies from peak power to zero.
The peak power = 2 x rms power.



Tutorial 11.07

11.07.1

Calculate ω:

500 rpm = 500 ÷ 60 = 8.33 Hz

ω = 2 × π × 8.33 = 52.4 rad s^{-1}

$\mathcal{E} = BAN\omega$ = 0.45 T × 6 ×10^{-4} m^2 × 10 turns × 52.4 rad s^{-1} = 0.142 V

V_{rms} = 0.142 V ÷ √2 = **0.10 V**

11.07.2

Power of the engine = 7 PS × 750 W PS^{-1} = 5250 W

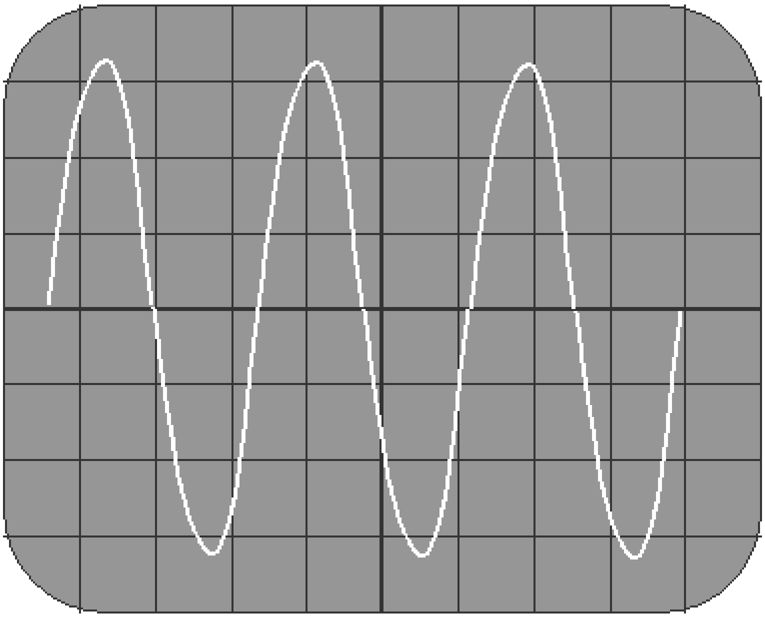
Efficiency = (3000 W ÷ 5250 W) × 100 % = **57 %** (Not very impressive)

If we measured the efficiency in terms of the chemical energy from the petrol, we would find that it was pitiful.

Tutorial 11.08

11.08.1

The CRO screen below shows a sinusoidal waveform:



The time base is set at 2 ms/cm and the y gain at 0.5 V/cm

(a) What is the peak to peak voltage?

The total height of the wave from peak to trough is 6.4 cm

$V_{pk\ to\ pk}$ = **3.2 V**

(b) What is the peak voltage?

V_{pk} = 3.2/2 = **1.6 V**

(c) What is the rms voltage?

$V_{rms} = V_{pk} /\sqrt{2}$ = 1.6 /$\sqrt{2}$ = 1.13 V

(d) What is the period?

1 cycle is 2.9 cm

Time period = 2.9 × 2 = 5.8 ms = **5.8×10^{-3} s**

(e) What is the frequency?

$f = 1/t$ = 1/5.8 × 10^{-3} = 178 Hz

11.08.2

(a)

There are 5 cycles over a distance of 10 cm.

Each cycle has a period represented by 2 cm.

Period = 2 cm × 2 ms s^{-1} = 4 × 10^{-3} s

$f = 1/T$ = (4 × 10^{-3} s)$^{-1}$ = **250 Hz**

(b)

Maximum voltage = +2 cm × 2.0 V cm^{-1} = **+4.0 V**

(c)

Minimum voltage = -2 cm × 2.0 V cm^{-1} = **-4.0 V**

(d)

This is an alternating waveform because the polarity is changing 250 times per second.

11.08.3

(a)

Maximum voltage = 4 cm × 2.0 V cm^{-1} = **+8.0 V**

(b)

Minimum voltage = 0 V

(c)

This is NOT an alternating waveform because the voltage changes from 0 V to + 8 V and back to 0 V. It is a unidirectional waveform.

Tutorial 11.09

11.09.1

(a)

Use $P=IV$.

$I = P \div V$ = 500 × 10^6 W ÷ 15000 V = **33 300 A**

(b)

The secondary current = 500 × 10^6 W ÷ 275000 V = **1820 A**

(c)

Turns ratio: assume that the primary is 1 turn.

$N_{prim}/N_{sec} = V_{prim}/V_{sec}$

$1/N_{sec}$ = 15000 ÷ 275000 = 5.45 × 10^{-2}

N_{sec} = 1/5.45 × 10^{-2} = **18.3**

Turns ratio Primary: Secondary = **1: 18.3**, or in whole numbers, **3: 55**

11.09.2

(a)

N_{prim}/N_{sec} = turns ratio

Turns ratio = 3600 ÷ 150 = **24:1**

(b)

Output voltage = 240 V ÷ 24 = 10 V

Output current = 1.5 A × 24 = **36 A**

11.09.3

The energy losses arise from:

- Resistance in the coils. If the currents are large, the power loss can become large, as the heating effect is governed by $P = I^2 R$.
- Eddy currents in the laminations of the core. These are reduced by lamination, but not stopped altogether.
- Work needs to be done to build up the magnetic field in the core. Not so much work is got out when the domains randomise again. This is called hysteresis.
- The magnetic material has a certain amount of remanence, which means that some domains are still lined up even when there is no current in the coil. This is true, even if the remanence is low, such as in soft iron.
- At a certain current level, all domains are lined up. The magnet is saturated and cannot become more magnetised. Therefore, a greater current will not pass across to the secondary coil.

11.09.4

(a)

Power of furnace = 1000 V × 40 000 A = **4.0×10^7 W**

(b)

Power required by the transformer = 4.0×10^7 W ÷ 0.95 = 4.21×10^7 W

Current = 4.21×10^7 W ÷ 1.32×10^5 V = 319 A = **320 A** (2 s.f.)

(c)

Power lost = 4.21×10^7 W - 4.0×10^7 W = 0.21×10^7 W = **2.1×10^6 W**

(d)

Such a transformer would need well-designed cooling systems which circulate oil. The oil is cooled in heat exchangers that transfer the heat to the air.

Tutorial 11.10

11.10.1

$I = P/V$ = 200 × 10^6 W ÷ 15000 V = 13 000 A

For each cable:

I = 13000 A ÷ 3 = **4300 A**

11.10.2

$P = I^2R$ = 4333 A^2 × 0.001 Ω = 1.88 × 10^4 W

Total loss = 1.88 × 10^4 W × 3 = **5.6 × 10^4 W**

11.10.3

Use $P=I^2R$ = 30 000 A^2 × 1.5 W = **1.35 × 10^9 W**

11.10.4

$P = I^2R$ = 1000 A^2 × 1.5 W = 1.5 × 10^6 W

11.10.5

(a)

Use $P=IV$. $I = P \div V$ = 500 × 10^6 W ÷ 15000 V = **33 300 A**

(b)

The secondary current = 500 × 10^6 W ÷ 275000 V = **1820 A**

(c)

Turns ratio: assume that the primary is 1 turn.

$N_{prim}/N_{sec} = V_{prim}/V_{sec}$

$1/N_{sec}$ = 15000 ÷ 275000 = 5.45 × 10^{-2}

N_{sec} = 1/5.45 × 10^{-2} = 18.3

Turns ratio Primary: Secondary = **1: 18.3**, or in whole numbers, **3: 55**

11.10.6

(a)

Resistance in the cable = 800 m × 0.045 Ω m^{-1} = **36 Ω**

(b)

$P = V^2/R \Rightarrow R = V^2/P$ = 230^2 ÷ 3000 W = **17.7 Ω**

(c)

R_{tot} = 36 Ω + 18 Ω = 54 Ω $\Rightarrow I = V/R$ = 230 V ÷ 54 Ω = **4.26 A**

(d)

$V = IR$ = 4.26 A × 17.7 Ω = **75 V**.

The machine would not work at all.

(e)

The new contractor would install a step up transformer, a high voltage transmission line, and a step-down transformer. The energy loss would be much less.

11.10.7

The engine can be assumed to give out its maximum power at top speed.

The time the battery will last for is:

t = 41 kWh ÷ 66 kW = **0.621 h** (= 37 minutes).

Range = 140 km h^{-1} × 0.621 h = **87 km**

(Your battery may be discharged, but the cops will ensure that you are fully charged:

- speeding (good for at least 3 points).
- reckless driving.
- failing to stop when required by a police officer.
- etc.)

11.10.8

Time taken = 41 kWh ÷ 3 kW = **13.6 h**

Tutorial 11.11

11.11.1

This is an incorrect statement.

Current cannot flow through a capacitor as the plates are separated by a layer of insulating material. Its resistance is infinite.

(If current were flowing through the capacitor, it would mean that the capacitor had been damaged.)

11.11.2

(a)

$$f = 1/2\pi X_C C$$

$f = 1 \div (2 \times \pi \times 320\ \Omega \times 10 \times 10^{-6}\ \text{F}) =$ **49.7 Hz** = 50 Hz (QED)

(b)

$X_C = 1 \div (2 \times \pi \times 1000\ \text{Hz} \times 10 \times 10^{-6}\text{F}) =$ **15.9 Ω**

11.11.3

The current is the same.

11.11.4

Current and voltage are always in phase in a resistive circuit.

11.11.5

It would be infinite.
A capacitor has infinite resistance as the dielectric is an insulator.

11.11.6

$$Z^2 = (20\ \Omega)^2 + (10\ \Omega)^2 = 500\ \Omega^2$$

$Z =$ **22.4 Ω**

Tutorial 11.12

11.12.1

RMS. It is the DC equivalent.

6 V rms will give the same brightness as 6 V DC.

11.12.2

$$L = \frac{4 \times \pi \times 10^{-7}\ \text{H m}^{-1} \times 1200^2 \times 4.0 \times 10^{-4}\ \text{m}^2}{0.050\ \text{m}} = \mathbf{\underline{0.014\ H}} = 14\ \text{mH}$$

11.12.3

$$12\ \text{V} = 2.3 \times 10^{-2}\ \text{H} \times dI/dt$$

$$dI/dt = 12\ \text{V} \div 2.3 \times 10^{-2}\ \text{H} = \mathbf{\underline{522\ A\ s^{-1}}}$$

11,12.4

(a)

$$\mathcal{E} = \tfrac{1}{2} LI^2 = \tfrac{1}{2} \times 10\ \text{H} \times 5\ (\text{A})^2 = \mathbf{\underline{125\ J}}$$

(b)

$$\mathcal{E} = -L(dI/dt) = (-)10\ \text{H} \times (5\ \text{A} \div 0.05\ \text{s}) = \mathbf{\underline{1000\ V}}$$

Tutorial 11.13

11.13.1

(a)

$$f = X_L/2\pi L$$

f = 320 Hz ÷ (2 × π × 10 × 10^{-3} H) = **5092 Hz** = 5100 Hz (QED)

(b)

X_L = 2 × π × 1000 Hz × 10 × 10^{-3} H = **62.8 ohm**

11.13.2

(a)

The two voltages are 90 degrees out of phase.

The total voltage is the vector sum.

(b)

V_R^2 = (3.5 V)^2 + (4.0 V)^2 = 28.25 V^2

V_R = **5.31 V**

11.13.3

Z^2 = (20 Ω)^2 + (10 Ω)^2 = 500 Ω^2

Z = **22.4 Ω**

11.13.4

(a)

P = (240 V)^2 ÷ 50 Ω = **<u>1150 W</u>**

(b)

$I^2 = P/R$ = 60 W ÷ 50 Ω = 1.2 A^2

I = **<u>1.10 A</u>**

(c)

$V = IR$ = 1.095 V × 50 Ω = **<u>54.8 V</u>** (= 55 V)

(d)

(240 V)^2 = (54.8 V)^2 + V_L^2

V_L^2 = (240 V)^2 - (54.8 V)^2 = 54600 V^2

V_L = 233.67 V = **<u>234 V</u>**

(e)

X_L = 234 V ÷ 1.095 A = 213.4 Ω = 213 Ω

(f)

$L = X_L/2\pi f$ = 213.4 Ω ÷ (2 × π × 50 Hz) = 0.679 H = **<u>0.68 H</u>** to 2 s.f.

(g)

Z^2 = (213.4 Ω)^2 + (50 Ω)^2 = 48040 Ω^2

Z = **<u>219 Ω</u>**

(h)

$V_{res} = IZ$ = 1.095 A × 219 Ω = **<u>240 V</u>**

The answer is consistent.